High School Algebra II

West Virginia math4life

Mathematics - High School Algebra II

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, rational, and radical functions. (In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.) Students will work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Polynomial, Rational, and Radical Relationships	**Trigonometric Functions**
• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)	• Apply knowledge of trigonometric functions to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)
Modeling with Functions	**Inferences and Conclusions from Data**
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.) • Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Polynomial, Rational, and Radical Relationships	
Perform arithmetic operations with complex numbers.	Standards 1-2
Use complex numbers in polynomial identities and equations.	Standards 3-5
Interpret the structure of expressions.	Standard 6-7
Write expressions in equivalent forms to solve problems.	Standard 8
Perform arithmetic operations on polynomials.	Standard 9
Understand the relationship between zeros and factors of polynomials.	Standard 10-11
Use polynomial identities to solve problems.	Standards 12-13
Rewrite rational expressions.	Standards 14-15
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 16
Represent and solve equations and inequalities graphically.	Standard 17
Analyze functions using different representations.	Standard 18
Trigonometric Functions	
Extend the domain of trigonometric functions using the unit circle.	Standards 19-20
Model periodic phenomena with trigonometric functions.	Standard 21
Prove and apply trigonometric identities.	Standard 22
Modeling with Functions	
Create equations that describe numbers or relationships.	Standards 23-26
Interpret functions that arise in applications in terms of a context.	Standards 27-29
Analyze functions using different representations.	Standards 30-32
Build a function that models a relationship between two quantities.	Standard 33
Build new functions from existing functions.	Standards 34-35
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 36
Inferences and Conclusions from Data	
Summarize, represent, and interpret data on a single count or measurement variable.	Standard 37
Understand and evaluate random processes underlying statistical experiments.	Standard 38-39
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Standards 40-43
Use probability to evaluate outcomes of decisions.	Standards 44-45

Cluster	**Perform arithmetic operations with complex numbers.**
M.A2HS.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
M.A2HS.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Cluster	**Use complex numbers in polynomial identities and equations.**
M.A2HS.3	Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to polynomials with real coefficients.
M.A2HS.4	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. Instructional Note: Limit to polynomials with real coefficients.
M.A2HS.5	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to polynomials with real coefficients.

Cluster	**Interpret the structure of expressions.**
M.A2HS.6	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Extend to polynomial and rational expressions.
M.A2HS.7	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions.

Cluster	**Write expressions in equivalent forms to solve problems.**
M.A2HS.8	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. Instructional Note: Consider extending this standard to infinite geometric series in curricular implementations of this course description.

Cluster	**Perform arithmetic operations on polynomials.**
M.A2HS.9	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Extend beyond the quadratic polynomials found in Algebra I.

Cluster	**Understand the relationship between zeros and factors of polynomials.**
M.A2HS.10	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by $x - a$ is p(a), so $p(a) = 0$ if and only if $(x - a)$ is a factor of p(x).
M.A2HS.11	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Cluster	Use polynomial identities to solve problems.
M.A2HS.12	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.
M.A2HS.13	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.

Cluster	Rewrite rational expressions.
M.A2HS.14	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. Instructional Note: The limitations on rational functions apply to the rational expressions.
M.A2HS.15	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials.

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.A2HS.16	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations.

Cluster	Represent and solve equations and inequalities graphically.
M.A2HS.17	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Instructional Note: Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Instructional Note: Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.

Cluster	**Analyze functions using different representations.**
M.A2HS.18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Instructional Note: Relate this standard to the relationship between zeros of quadratic functions and their factored forms.

Trigonometric Functions

Cluster	**Extend the domain of trigonometric functions using the unit circle.**
M.A2HS.19	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
M.A2HS.20	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Cluster	**Model periodic phenomena with trigonometric functions.**
M.A2HS.21	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Cluster	**Prove and apply trigonometric identities.**
M.A2HS.22	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. Instructional Note: An Algebra II course with an additional focus on trigonometry could include the standard "Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems." This could be limited to acute angles in Algebra II.

Modeling with Functions

Cluster	**Create equations that describe numbers or relationships.**
M.A2HS.23	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
M.A2HS.24	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. (e.g., Finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line).

M.A2HS.25	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.
M.A2HS.26	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.
Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A2HS.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Emphasize the selection of a model function based on behavior of data and context.
M.A2HS.28	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Note: Emphasize the selection of a model function based on behavior of data and context.
M.A2HS.29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Note: Emphasize the selection of a model function based on behavior of data and context.
Cluster	**Analyze functions using different representations.**
M.A2HS.30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.

M.A2HS.31	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.A2HS.32	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
Cluster	**Build a function that models a relationship between two quantities.**
M.A2HS.33	Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Develop models for more complex or sophisticated situations than in previous courses.
Cluster	**Build new functions from existing functions.**
M.A2HS.34	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types.
M.A2HS.35	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (e.g., $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.) Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Extend this standard to simple rational, simple radical, and simple exponential functions; connect this standard to M.A2HS.34.
Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.A2HS.36	For exponential models, express as a logarithm the solution to $a b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Instructional Note: Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x +log y.

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.A2HS.37	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.

Cluster	**Understand and evaluate random processes underlying statistical experiments.**
M.A2HS.38	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Instructional Note: Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.
M.A2HS.39	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. (e.g., A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?)

Cluster	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**
M.A2HS.40	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.
M.A2HS.41	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.

M.A2HS.42	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.A2HS.43	Evaluate reports based on data. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

Cluster	**Use probability to evaluate outcomes of decisions.**
M.A2HS.44	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.
M.A2HS.45	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.