Mathematics – High School Mathematics I

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All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students in this course will focus on six critical units that deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Students in Mathematics 1 will use properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades and develop connections between the algebraic and geometric ideas studied. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., “The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer?”; “Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?”)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie’s apple picking?)
Reasoning with Equations	**Descriptive Statistics**
• Translate between various forms of linear equations. (e.g., The perimeter of a rectangle is given by $P = 2W + 2L$. Solve for W and restate in words the meaning of this new formula in terms of the meaning of the other variables.) • Explore systems of equations, find and interpret their solutions. (e.g., The high school is putting on the musical Footloose. The auditorium has 300 seats. Student tickets are $3 and adult tickets are $5. The royalty for the musical is $1300. What combination of student and adult tickets do you need to fill the house and pay the royalty? How could you change the price of tickets so more students can go?)	• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student’s university GPA based on his high school GPA.)
Congruence, Proof, and Constructions	**Connecting Algebra and Geometry through Coordinates**
• Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Linear and Exponential Relationships	
Represent and solve equations and inequalities graphically.	Standards 9-11
Understand the concept of a function and use function notation.	Standards 12-14
Interpret functions that arise in applications in terms of a context.	Standards 15-17
Analyze functions using different representations.	Standards 18-19
Build a function that models a relationship between two quantities.	Standards 20-21
Build new functions from existing functions.	Standards 22
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 23-25
Interpret expressions for functions in terms of the situation they model.	Standard 26
Reasoning with Equations	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 27
Solve equations and inequalities in one variable.	Standard 28
Solve systems of equations.	Standards 29-30
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 31-33
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 34-35
Interpret linear models.	Standards 36-38
Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 39-43
Understand congruence in terms of rigid motions.	Standards 44-46
Make geometric constructions.	Standards 47-48
Connecting Algebra and Geometry through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 49-51

Cluster	Reason quantitatively and use units to solve problems.
M.1HS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.1HS.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.1HS.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.1HS.4	Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.1HS.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.1HS.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.1HS.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.
M.1HS.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Cluster	Represent and solve equations and inequalities graphically.
M.1HS.9	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.1HS.10	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value exponential, and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.
M.1HS.11	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	Understand the concept of a function and use function notation.
M.1HS.12	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.
M.1HS.13	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.

M.1HS.14	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions. Draw connection to M.1HS.21, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.1HS.15	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.1HS.16	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.
M.1HS.17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.

Cluster	**Analyze functions using different representations.**
M.1HS.18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.
M.1HS.19	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.

Cluster	**Build a function that models a relationship between two quantities.**
M.1HS.20	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.
M.1HS.21	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	**Build new functions from existing functions.**
M.1HS.22	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.1HS.23	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.1HS.24	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
M.1HS.25	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.

Cluster	**Interpret expressions for functions in terms of the situation they model.**
M.1HS.26	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Reasoning with Equations

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.1HS.27	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.

Cluster	**Solve equations and inequalities in one variable.**
M.1HS.28	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Cluster	**Solve systems of equations.**
M.1HS.29	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M.1HS.50, which requires students to prove the slope criteria for parallel lines.
M.1HS.30	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M.1HS.50, which requires students to prove the slope criteria for parallel lines.

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.1HS.31	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.1HS.32	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.1HS.33	Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.1HS.34	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.
M.1HS.35	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. (Focus should be on situations for which linear models are appropriate.) c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	Interpret linear models.
M.1HS.36	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.1HS.37	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.

M.1HS.38	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.

Congruence, Proof, and Constructions

Cluster	**Experiment with transformations in the plane.**
M.1HS.39	Know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.1HS.40	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS.41	Given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS.42	Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS.43	Given a geometric figure and a rotation, reflection or translation draw the transformed figure using, e.g., graph paper, tracing paper or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).

Cluster	Understand congruence in terms of rigid motions.
M.1HS.44	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.1HS.45	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.1HS.46	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Cluster	Make geometric constructions.
M.1HS.47	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.
M.1HS.48	Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.1HS.49	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles).
M.1HS.50	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). Relate work on parallel lines to work on M.1HS.29 involving systems of equations having no solution or infinitely many solutions.
M.1HS.51	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula). Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). This standard provides practice with the distance formula and its connection with the Pythagorean theorem.