

Mathematics – Grade 6

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the sixth grade will focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting and using expressions and equations; and (4) developing understanding of statistical thinking. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in sixth grade will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from fifth grade, the following chart represents the mathematical understandings that will be developed in sixth grade:



<p>Ratios and Proportional Reasoning</p> <ul style="list-style-type: none"> Understand ratios and rates, and solve problems involving proportional relationships (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours?). 	<p>The Number System</p> <ul style="list-style-type: none"> Divide fractions and solve related word problems (e.g., How wide is a rectangular strip of land with length $\frac{3}{4}$ mile and area $\frac{1}{2}$ square mile?). Use positive and negative numbers together to describe quantities; understand the ordering and absolute values of positive and negative numbers.
<p>Expressions and Equations</p> <ul style="list-style-type: none"> Work with variables and expressions by generalizing the way numbers work (e.g., When adding numbers, the order doesn't matter, so $x + y = y + x$; likewise, properties of addition and multiplication can be used to rewrite $24x + 18y$ as $6(4x + 3y)$, or $y + y + y$ as $3y$). Write equations to solve word problems and describe relationships between quantities (e.g., The distance D traveled by a train in time T might be expressed by an equation $D = 85T$, where D is in miles and T is in hours.). 	<p>Geometry</p> <ul style="list-style-type: none"> Reason about relationships between shapes to determine area, surface area, and volume.
<p>Statistics and Probability</p>	
<ul style="list-style-type: none"> Create graphical representations of data and reason about statistical distributions. 	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

<p>Ratios and Proportional Relationships</p>	
<p>Understand ratio concepts and use ratio reasoning to solve problems.</p>	<p>Standards 1-3</p>
<p>The Number System</p>	
<p>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</p>	<p>Standard 4</p>
<p>Compute fluently with multi-digit numbers and find common factors and multiples.</p>	<p>Standards 5-7</p>
<p>Apply and extend previous understandings of numbers to the system of rational numbers.</p>	<p>Standards 8-11</p>

Expressions and Equations	
Apply and extend previous understandings of arithmetic to algebraic expressions.	Standards 12-15
Reason about and solve one-variable equations and inequalities.	Standards 16-19
Represent and analyze quantitative relationships between dependent and independent variables.	Standard 20
Geometry	
Solve real-world and mathematical problems involving area, surface area, and volume.	Standards 21-24
Statistics and Probability	
Develop understanding of statistical variability.	Standards 25-27
Summarize and describe distributions.	Standards 28-29

Ratios and Proportional Relationships

Cluster	Understand ratio concepts and use ratio reasoning to solve problems.
M.6.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. (e.g., “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”)
M.6.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. (e.g., “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”) Instructional Note: Expectations for unit rates in this grade are limited to non-complex fractions.
M.6.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. <ul style="list-style-type: none"> a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?) c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Cluster	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
M.6.4	Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem. (e.g., Create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $3/4$ -cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?)
Cluster	Compute fluently with multi-digit numbers and find common factors and multiples.
M.6.5	Fluently divide multi-digit numbers using the standard algorithm.
M.6.6	Fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.
M.6.7	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor (e.g., express $36 + 8$ as $4(9 + 2)$).
Cluster	Apply and extend previous understandings of numbers to the system of rational numbers.
M.6.8	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
M.6.9	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. <ul style="list-style-type: none"> a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

M.6.10	<p>Understand ordering and absolute value of rational numbers.</p> <ol style="list-style-type: none"> Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. (e.g., interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.) Write, interpret, and explain statements of order for rational numbers in real-world contexts (e.g., write $-3^\circ \text{C} > -7^\circ \text{C}$ to express the fact that -3°C is warmer than -7°C). Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. (e.g., for an account balance of -30 dollars, write $-30 = 30$ to describe the size of the debt in dollars.) Distinguish comparisons of absolute value from statements about order. (e.g., recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.)
M.6.11	<p>Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>

Expressions and Equations

Cluster	Apply and extend previous understandings of arithmetic to algebraic expressions.
M.6.12	Write and evaluate numerical expressions involving whole-number exponents.
M.6.13	<p>Write, read and evaluate expressions in which letters stand for numbers.</p> <ol style="list-style-type: none"> Write expressions that record operations with numbers and with letters standing for numbers. (e.g., Express the calculation, “Subtract y from 5” as $5 - y$.) Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. (e.g., Describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.) Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order: Order of Operations (e.g., use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$).
M.6.14	Apply the properties of operations to generate equivalent expressions (e.g., apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$).

M.6.15	Identify when two expressions are equivalent; i.e., when the two expressions name the same number regardless of which value is substituted into them. (e.g., The expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.)
Cluster	Reason about and solve one-variable equations and inequalities.
M.6.16	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
M.6.17	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or depending on the purpose at hand, any number in a specified set.
M.6.18	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.
M.6.19	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Cluster	Represent and analyze quantitative relationships between dependent and independent variables.
M.6.20	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (e.g., In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.)

Geometry

Cluster	Solve real-world and mathematical problems involving area, surface area, and volume.
M.6.21	Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
M.6.22	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

M.6.23	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
M.6.24	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability

Cluster	Develop understanding of statistical variability.
M.6.25	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. (e.g., “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.)
M.6.26	Through informal observation, understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean/median), spread (range), and overall shape.
M.6.27	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number.
Cluster	Summarize and describe distributions.
M.6.28	Display numerical data in plots on a number line, including dot plots, histograms and box plots.
M.6.29	Summarize numerical data sets in relation to their context, such as by: <ul style="list-style-type: none"> a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.