Mathematics – STEM Readiness

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. This course is designed for students who have completed the Math III (LA) course and subsequently decided they are interested in pursuing a STEM career. It includes standards that would have been covered in Mathematics III (STEM) but not in Mathematics III (LA) (i.e. standards that are marked with a "+"), selected topics from the Mathematics IV course, and topics drawing from standards covered in Mathematics I and Mathematics II as needed for coherence. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

<table>
<tr><th>Arithmetic and Algebra of Complex Numbers</th><th>Polynomial, Rational, and Radical Relationships</th></tr>
<tr><td>• Understand that the arithmetic and algebra of expressions involving rational numbers is governed by the same rules as the arithmetic and algebra of real numbers.</td><td>• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)</td></tr>
<tr><th>Probability for Decisions</th><th>Trigonometry of General Triangles</th></tr>
<tr><td>• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</td><td rowspan="3">• Apply knowledge of the Law of Sines and the Law of Cosines to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)</td></tr>
<tr><th>Functions and Modeling</th></tr>
<tr><td>• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.)</td></tr>
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Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Arithmetic and Algebra of Complex Numbers	
Perform arithmetic operations with complex numbers.	Standards 1-3
Represent complex numbers and their operations on the complex plane.	Standards 4-6
Use complex numbers in polynomial identities and equations.	Standards 7-9
Polynomial, Rational, and Radical Relationships	
Use polynomial identities to solve problems.	Standard 10
Rewrite rational expressions.	Standard 11
Probability for Decisions	
Use probability to evaluate outcomes of decisions.	Standards 12-13
Trigonometry of General Triangles	
Apply trigonometry to general triangles.	Standards 14-16
Functions and Modeling	
Analyze functions using different representations.	Standards 17-19
Building a function that models a relationship between two quantities.	Standards 20-21
Build new functions from existing functions.	Standards 22-26
Extend the domain of trigonometric functions using the unit circle.	Standards 27-28
Model periodic phenomena using trigonometric functions.	Standards 29-30
Prove and apply trigonometric identities.	Standard 31

Arithmetic and Algebra of Complex Numbers

Cluster	**Perform arithmetic operations with complex numbers**
M.SRM.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
M.SRM.2	Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers.
M.SRM.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Cluster	**Represent complex numbers and their operations on the complex plane**
M.SRM.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.

M.SRM.5	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. (e.g., $(-1 + \sqrt{3}\ i)^3 = 8$ because $(-1 + \sqrt{3}\ i)$ has modulus 2 and argument 120°.)
M.SRM.6	Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Cluster	**Use complex numbers in polynomial identities and equations**
M.SRM.7	Solve quadratic equations with real coefficients that have complex solutions.
M.SRM.8	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
M.SRM.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Polynomials, Rational, and Radical Relationships

Cluster	**Use polynomial identities to solve problems.**
M.SRM.10	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Cluster	**Rewrite rational expressions**
M.SRM.11	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

Probability for Decisions

Cluster	**Use probability to evaluate outcomes of decisions.**
M.SRM.12	Use probabilities to make fair decisions (e.g. drawing by lot or using a random number generator).
M.SRM.13	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).

Trigonometry of General Triangles

Cluster	**Apply trigonometry to general triangles.**
M.SRM.14	Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.SRM.15	Prove the Laws of Sines and Cosines and use them to solve problems.

M.SRM.16	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems or resultant forces).

Functions and Modeling

Cluster	**Analyze functions using different representations.**
M.SRM.17	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
M.SRM.18	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.
M.SRM.19	Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline, and amplitude.
Cluster	**Building a function that models a relationship between two quantities.**
M.SRM.20	Write a function that describes a relationship between two quantities.
M.SRM.21	Compose functions. (e.g., If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.)
Cluster	**Build new functions from existing functions.**
M.SRM.22	Find inverse functions.
M.SRM.23	Verify by composition that one function is the inverse of another.
M.SRM.24	Read values of an inverse function from a graph or a table, given that the function has an inverse.
M.SRM.25	Produce an invertible function from a non-invertible function by restricting the domain.
M.SRM.26	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
Cluster	**Extend the domain of trigonometric functions using the unit circle.**
M.SRM.27	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
M.SRM.28	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Cluster	**Model periodic phenomena using trigonometric functions.**
M.SRM.29	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
M.SRM.30	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Cluster	**Prove and apply trigonometric identities.**
M.SRM.31	Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems.