Transition Mathematics for Seniors

Mathematics – Transition Mathematics for Seniors

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Transition Mathematics for Seniors prepares students for their entry-level credit-bearing liberal studies mathematics course at the post-secondary level. Students will solidify their quantitative literacy by enhancing numeracy and problem solving skills as they investigate and use the fundamental concepts of algebra, geometry, and introductory trigonometry. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity: The Real Number System The Complex Number System	**Algebra:** Seeing Structure in Expressions Arithmetic with Polynomials and Rational Expressions Creating Equations Reasoning with Equations and Inequalities
• Develop an understanding of basic operations, equivalent representations, and properties of the real and complex number systems.	• Create equations or inequalities that model physical situations. • Solve systems of equations, with an emphasis on efficiency of solution as well as reasonableness of answers, given physical limitations.
Functions: Interpreting Functions Building Functions	**Geometry:** Geometric Measuring and Dimension Expressing Geometric Properties with Equations Modeling with Geometry
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent and interpret functions and their applications.	• Use coordinates and to prove geometric properties algebraically.
Statistics and Probability: Interpreting Categorical and Quantitative Data Making Inferences and Justifying Conclusions	
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Number and Quantity – The Real Number System	
Extend the properties of exponents to rational exponents.	Standard 1-2
Number and Quantity – The Complex Number System	
Use complex numbers in polynomial identities and equations.	Standard 3
Algebra – Seeing Structure in Expressions	
Interpret the structure of expressions.	Standard 4
Write expressions in equivalent forms to solve problems.	Standards 5-6
Understand the connections between proportional relationship, lines, and linear equations.	Standards 7-9

Algebra – Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	Standard 10
Algebra – Creating Equations	
Create equations that describe numbers or relationships.	Standards 11-14
Algebra – Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 15
Solve equations and inequalities in one variable.	Standards 16-18
Solve systems of equations.	Standards 19-21
Represent and solve equations and inequalities graphically.	Standards 22-23
Functions – Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 24
Interpret functions that arise in applications in terms of the context.	Standards 25-28
Analyze functions using different representations.	Standards 29-35
Functions – Building Functions	
Build a function that models a relationship between two quantities.	Standards 36-37
Geometry – Geometric Measuring and Dimension	
Explain volume formulas and use them to solve problems.	Standards 38-39
Geometry – Expressing Geometric Properties with Equations	
Use coordinates to prove simple geometric theorems algebraically.	Standards 40-41
Geometry – Modeling with Geometry	
Apply geometric concepts in modeling situations.	Standard 42
Statistics and Probability – Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 43-46
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 47-51
Statistics and Probability – Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments.	Standard 52

Number and Quantity – The Real Number System

Cluster	**Extend the properties of exponents to rational exponents.**
M.TMS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

M.TMS.2	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Number and Quantity – The Complex Number System

Cluster	**Use complex numbers in polynomial identities and equations.**
M.TMS.3	Solve quadratic equations with real coefficients that have complex solutions.

Algebra – Seeing Structure in Expressions

Cluster	**Interpret the structure of expressions.**
M.TMS.4	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Cluster	**Write expressions in equivalent forms to solve problems.**
M.TMS.5	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
M.TMS.6	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Cluster	**Understand the connections between proportional relationship, lines, and linear equations.**
M.TMS.7	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
M.TMS.8	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plan; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.
M.MTS.9	Solve linear equations in one variable.

Algebra – Arithmetic with Polynomials and Rational Expressions

Cluster	**Perform arithmetic operations on polynomials.**
M.TMS.10	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.

Algebra – Creating Equations

Cluster	**Create equations that describe numbers or relationships.**
M.TMS.11	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
M.TMS.12	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
M.TMS.13	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
M.TMS.14	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Algebra – Reasoning with Equations and Inequalities

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.TMS.15	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Cluster	**Solve equations and inequalities in one variable.**
M.TMS.16	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
M.TMS.17	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
M.TMS.18	Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Cluster	Solve systems of equations.
M.TMS.19	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
M.TMS.20	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
M.TMS.21	Explain why the x-coordinates of the points where the graphs of the equation y = f(x) and y = g(x) intersect are the solution of the equation f(x) = g (x); find the solution approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).

Cluster	Represent and solve equations and inequalities graphically.
M.TMS.22	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
M.TMS.23	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions – Interpreting Functions

Cluster	Understand the concept of a function and use function notation.
M.TMS.24	Understand a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

Cluster	Interpret functions that arise in applications in terms of the context.
M.TMS.25	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
M.TMS.26	Interpret the parameters in a linear or exponential function in terms of a context.
M.TMS.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
M.TMS.28	Distinguish between situations that can be modeled with linear functions and with exponential functions.

Cluster	Analyze functions using different representations.
M.TMS.29	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line, give examples of functions that are not linear.

M.TMS.30	Describe qualitatively the functional relationship between two quantities by analyzing a graph.
M.TMS.31	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
M.TMS.32	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
M.TMS.33	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasingly linearly, quadratically, or (more generally) as a polynomial function.
M.TMS.34	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
M.TMS.35	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Functions - Building Functions

Cluster	**Build a function that models a relationship between two quantities.**
M.TMS.36	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
M.TMS.37	Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Geometry – Geometric Measuring and Dimension

Cluster	**Explain volume formulas and use them to solve problems.**
M.TMS.38	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

M.TMS.39	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Geometry – Expressing Geometric Properties with Equations

Cluster	**Use coordinates to prove simple geometric theorems algebraically**
M.TMS.40	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
M.TMS.41	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula).

Geometry – Modeling with Geometry

Cluster	**Apply geometric concepts in modeling situations.**
M.TMS.42	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**
M.TMS.43	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Interpret linear models.
M.TMS.44	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
M.TMS.45	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.TMS.46	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.TMS.47	Represent data with plots on the real number line (dot plots, histograms, and box plots).

M.TMS.48	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
M.TMS.49	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.TMS.50	Computer (using technology) and interpret the correlation coefficient of a linear fit.
M.TMS.51	Distinguish between correlation and causation.

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	**Understand and evaluate random processes underlying statistical experiments**
M.TMS.52	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.