

# Content and Timeline for Mathematics

Mathematics IV
Trigonometry/Pre-calculus





### West Virginia Board of Education 2021-2022

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# **Mathematics IV-Trigonometry/Pre-calculus**

The West Virginia College- and Career-Readiness Standards for mathematics emphasize key content, skills, and Mathematical Habits of Mind at each grade level. The focus of instruction is placed on course standards; instruction should integrate content standards and Mathematical Habits of Mind. Instruction should be attentive to learning across course domains and link major topics within domains. Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Students in Mathematics IV-Trigonometry/Pre-calculus will generalize and use abstract learning accumulated through previous courses as the final springboard to calculus. Students will take an extensive look at the relationships among complex numbers, vectors, and matrices. They will build on their understanding of functions, analyze rational functions using an intuitive approach to limits and synthesize functions by considering compositions and inverses. Students will expand their work with trigonometric functions and their inverses and complete the study of conic sections begun in previous courses. They will enhance their understanding of probability by considering probability distributions and have previous experiences with series augmented. Students will continue developing mathematical proficiency in a developmentally appropriate progression of standards.

The table below highlights the content at the cluster level for Mathematics IV-Trigonometry/Pre-calculus standards. The bulk of instructional time should be given to the clusters and the standards within them. Standards should not be neglected; to do so would result in gaps in students' learning, including skills and understandings they may need in later grades. Instruction should reinforce standards within the clusters by including problems and activities that support natural connections between clusters. **Teachers and administrators alike should note that the standards are not topics to be checked off after being covered in isolated units of instruction;** rather, they provide content to be developed throughout the school year through rich instructional experiences presented in a coherent manner.

### **Connections in the Integrated and Traditional Pathway**

Students in Mathematics IV-Trigonometry/Precalculus extend their work with complex numbers, which started in Mathematics III or Algebra II, to see that complex numbers can be represented in the Cartesian plane and that operations with complex numbers have a geometric interpretation.

Students begin working with vectors, representing them geometrically and performing operations with them. They connect the notion of vectors to complex numbers. Students also work with matrices and their operations, experiencing for the first time an algebraic system in which multiplication is not commutative. Additionally, they see the connection between matrices and transformations of the plane—namely, that a vector in the plane can be multiplied by a 2 x 2 matrix to produce another vector—and they work with matrices from the perspective of transformations. They also find inverse matrices and use matrices to represent and solve linear systems.

Students extend their work with trigonometric functions, investigating the reciprocal functions **secant**, **cosecant**, and **cotangent** and the graphs and properties associated with those functions. Students find inverse trigonometric functions by appropriately restricting the domains of the standard trigonometric functions and use them to solve problems that arise in modeling contexts.

Although students in Mathematics IV-Trigonometry/Pre-calculus have worked previously with parabolas and circles, they now work with ellipses and hyperbolas.

Finally, students work with rational functions that are more complicated, graphing them and determining zeros, y-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points.

Mathematics IV-Trigonometry/Pre-calculus	West Virginia College- and Career-Readiness Standards						
Building Relationships among Complex Numbers, Vectors, and Matrices							
Clusters							
» Perform arithmetic operations with complex numbers.	» M.4HSTP.1						
» Represent complex numbers and their operations on the complex plane.	» M.4HSTP.2-4						
» Represent and model with vector quantities.	» M.4HSTP.5-7						
» Perform operations on vectors.	» M.4HSTP.8-9						
» Perform operations on matrices and use matrices in applications.	» M.4HSTP.10-16						
» Solve systems of equations	» M.4HSTP.17-18						
Analysis and Synthesis of Functions							
Clusters							
Analyze functions using different representations.	» M.4HSTP.19						
» Build a function that models a relationship between two quantities.	» M.4HSTP.20						
» Build new functions from existing functions.	» M.4HSTP.21-22						
Trigonometric and Inverse Trigonometric Functions of Real Numbers							
Clusters							
» Extend the domain of trigonometric functions using the unit circle.	» M.4HSTP.23-24						
» Model periodic phenomena with trigonometric functions.	» M.4HSTP.25-27						
» Prove and apply trigonometric identities.	» M.4HSTP.28						
» Apply transformations of function to trigonometric functions.	» M.4HSTP.29						
Derivations in Analytic Geometry							
Clusters							
» Translate between the geometry description and the equation for a conic	» M.4HSTP.30						
section.							
» Explain volume formulas and use them to solve problems.	» M.4HSTP.31						

Mathematics IV-Trigonometry/Pre-calculus	West Virginia College- and Career-Readiness Standards							
Modeling with Probability								
<ul><li>Clusters</li><li>Calculate expected values and use them to solve problems.</li><li>Use probability to evaluate outcomes of decisions.</li></ul>	<ul><li>» M.4HSTP.32-35</li><li>» M.4HSTP.36</li></ul>							
Series and Informal Limits								
<ul><li>Clusters</li><li>When the sum of the</li></ul>	<ul><li>» M.4HSTP.37-38</li><li>» M.4HSTP.39-40</li></ul>							

### **Explanations**

- **Domains** are broad components that make up a content area. Domains in mathematics vary by grade-level and by course. For example, the six domains for mathematics of Mathematics IV-Trigonometry/Pre-calculus are Building Relationships among Complex Numbers, Vectors, and Matrices; Analysis and Synthesis of Functions; Trigonometric and Inverse Trigonometric Functions of Real Numbers; Derivations in Analytic Geometry; Modeling with Probability; and Series and Informal Limits.
- **» Clusters** are groups of standards that define the expectations students must demonstrate to be college- and career-ready.
- **» Standards** are expectations for what students should know, understand and be able to do; standards represent educational goals.

Note of caution: Neglecting material will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades.

# **Mathematics IV-Trigonometry/Pre-calculus Content Plan**

Teachers must provide students the opportunity to master each of the course content standards. It is important to understand that neglecting grade-level or course content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in this and later courses. Any content plan must demonstrate a means by which students can be provided the opportunity to address all course content standards and to revisit and practice skills and strengthen understandings throughout the school year.

DOMAIN TOPIC	Trigonometric and Inverse Trigonometric Functions of Real Numbers Unit Circle, Modeling Periodic Behavior	Trigonometric and Inverse Trigonometric Functions of Real Numbers Trigonometric Functions, Equations, Identities	Analysis and Synthesis of Functions Rational Functions and Limits	Analysis and Synthesis of Functions Exponential, Logarithms, Inverse Functions	Series and Informal Limits Sigma Notation, Geometric Series	Building Relationships among Complex Numbers, Vectors, and Matrices Complex Numbers and Vectors	Building Relationships among Complex Numbers, Vectors, and Matrices Vectors and Matrices	Derivations in Analytic Geometry Conic Sections	Modeling with Probability Probability
SAMPLE	August/ September	September/ October	October/ November	November/ December	January	February	March	April	May/June
CONTENT STANDARDS	M.4HSTP.23 M.4HSTP.24 M.4HSTP.25	M.4HSTP.26 M.4HSTP.27 M.4HSTP.28 M.4HSTP.29	M.4HSTP.19 M.4HSTP.20	M.4HSTP.21 M.4HSTP.22	M.4HSTP.37 M.4HSTP.38 M.4HSTP.39 M.4HSTP.40	M.4HSTP:1 M.4HSTP:2 M.4HSTP:3 M.4HSTP:4 M.4HSTP:5 M.4HSTP:6 M.4HSTP:7	M.4HSTP.8 M.4HSTP.9 M.4HSTP.10 M.4HSTP.11 M.4HSTP.12 M.4HSTP.13 M.4HSTP.14 M.4HSTP.15 M.4HSTP.16 M.4HSTP.17 M.4HSTP.17	M.4HSTP.30 M.4HSTP.31	M.4HSTP.32 M.4HSTP.33 M.4HSTP.34 M.4HSTP.35 M.4HSTP.36

ATIONALE

In the sample above, Mathematics IV-Trigonometry/Pre-calculus begins with students reasoning about trigonometric functions and their inverses and extending this understanding to rational, exponential, and logarithmic functions then to explore series, complex numbers, vectors, and matrices. This format starts with a mathematical concept that is recognized by students and incorporates opportunities to practice concepts and skills developed in prior years. In the plan, students can be provided opportunities to strengthen their ability in procedural fluency, conceptual understanding, and applications.



W. Clayton Burch West Virginia Superintendent of Schools