MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Applied Statistics

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Applied Statistics provides authentic experiences in statistics designed to strengthen students' application of the statistical method. Students will conduct statistical simulations to model everyday situations in an increasingly data-rich world. Students in this course will select appropriate graphical and numerical methods to explore data, design and implement a plan to collect and analyze data, and use probability to evaluate outcomes and make decisions. Students will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, radical, and rational functions. Students will use multiple representations, technology, applications and modeling in problem-solving contexts. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Exploring Data	Designing Studies
• Represent data visually and calculate statistical measures that describe the data set. (e.g., Construct and interpret a histogram for a student created data set.)	• Design a plan to collect data using an appropriate sampling method to solve a problem. (e.g., Design and conduct an experiment to determine the effect of a treatment.)
Functions and Modeling	**Probability and Informed Decisions**
• Explore expressions, functions, and models to highlight key features that provide insight into their structure, properties, and form. (e.g., Model situations using mathematics to provide an optimal solution.)	• Make inferences and justify conclusions from data and analyze decisions and strategies using probability concepts. (e.g., Compare experimental and theoretical probabilities to make informed decisions.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Applied Statistics:

Exploring Data	
Select appropriate graphical and numerical methods to explore data.	Standards 1-7
Designing Studies	
Design and implement a plan to collect and analyze data.	Standards 8-12
Functions and Modeling	
Explore expressions, functions, and models to describe numbers or relationships.	Standards 13-21
Probability and Informed Decisions	
Use probability to evaluate outcomes and make decisions.	Standards 22-29

Exploring Data

Cluster	Select appropriate graphical and numerical methods to explore data.
M.ASHS.1	Generate appropriate ways to display various types of data. Instructional Note: Build on data displays introduced in prior courses.
M.ASHS.2	Calculate appropriate measures of center, variability, and position for data. Instructional note: Include comparisons of mean vs. median, standard deviation vs. IQR.
M.ASHS.3	Use graphical displays and summary statistics to make conclusions. Informally develop the concept of statistical significance; a result that is unlikely to have occurred by chance alone. Instructional Note: Focus on statistics as a way of dealing with, not eliminating, inherent randomness.

Cluster	Select appropriate graphical and numerical methods to explore data.
M.ASHS.4	Represent data in two variables to model relationships between quantities. Instructional Note: Students will use multiple representations with appropriate labels and scales.
M.ASHS.5	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities. Instructional Note: Focus on form, strength, direction, and departures from a model based on data and context.
M.ASHS.6	Compare characteristics of two data sets each represented in different ways (algebraically, graphically, numerically, and verbally). Instructional Note: Focus on applications and how key features relate to characteristics of a situation and select an appropriate model.
M.ASHS.7	Use appropriate measures of center and spread to describe a distribution. Instructional Note: Emphasize that only some data are well described by a normal distribution.

Designing Studies

Cluster	Design and implement a plan to collect and analyze data.
M.ASHS.8	Develop a process for making inferences about population parameters based on a random sample through data collection and analysis.
M.ASHS.9	Evaluate the results from a given data-generating process to determine consistency between theoretical and experimental probabilities. Instructional Note: Include the Law of Large Numbers.
M.ASHS.10	Recognize the purposes of and differences among sample surveys, experiments, and observational studies. Explain the importance of randomization in each method. Instructional Note: Emphasize that the way in which data is collected determines the scope and nature of the conclusions.
M.ASHS.11	Use data from a sample survey to estimate a population mean or proportion. Instructional Note: Develop the connection between sample size and margin of error.
M.ASHS.12	Design and conduct an experiment to compare two treatments. Instructional Note: Include randomization, replication, blocking, and control in the design.

Functions and Modeling

Cluster	Explore expressions, functions, and models to describe numbers or relationships.
M.ASHS.13	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, simple rational and exponential functions.
M.ASHS.14	Develop the concept of a complex number i such that $i^2 = -1$. Understand that every complex number can be written in the form a + bi with a and b real.

Cluster	Explore expressions, functions, and models to describe numbers or relationships.
M.ASHS.15	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
M.ASHS.16	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions.
M.ASHS.17	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
M.ASHS.18	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials.
M.ASHS.19	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations.
M.ASHS.20	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., solve $z = \frac{x-\mu}{\sigma}$ for σ and Margin of Error = $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ for n.) Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.
M.ASHS.21	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Include intercepts, intervals where the function is increasing, decreasing, positive, negative, relative extrema, symmetries, and end behavior. Instructional Note: Emphasize the selection of a model function based on the behavior of data in context.

Probability and Informed Decisions

Cluster	Use probability to evaluate outcomes and make decisions.
M.ASHS.22	Connect sampling variability and margin of error to generate and interpret plausible parameter values. Instructional Note: The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred by chance alone. Focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.ASHS.23	Interpret results from a randomized experiment comparing two treatments. Use simulations to decide if experimental results are significant. Instructional Note: Develop informally the comparison of an observed result and an established probability value (for example $p \leq 0.05$).
M.ASHS.24	Evaluate claims based on data reports. Instructional Note: Data reports can be gathered from media.

Cluster	Use probability to evaluate outcomes and make decisions.
M.ASHS.25	Use probability rules to make fair decisions. Instructional Note: Extend and apply probability rules introduced in prior courses to more complex probability models that involve decisions. Include examples that yield both false positive and false negative results.
M.ASHS.26	Use two-way tables, tree diagrams, Venn diagrams, or 10 x 10 grids to model probabilities.
M.ASHS.27	Justify a decision using probability rules (e.g., product testing, medical testing, weather forecasting, marketing, or sports coaching decisions). Instructional Note: Extend and apply probability rules introduced in prior courses to more complex probability models that involve decisions. Include examples that yield both false positive and false negative results.
M.ASHS.28	Perform appropriate calculations for given outcomes and decisions based on expected values for non-normal distributions. Instructional Note: Focus on uniform, discrete, continuous (geometric areas), or games of chance.
M.ASHS.29	Given data from a normal distribution, use the mean and standard deviation to estimate population percentages. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Recognize that there are data sets for which such a procedure is not appropriate. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities).