

MHM7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

MHM8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Financial Algebra/Mathematics

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students in this course will focus on financial applications designed to deepen and extend understanding of mathematics. Students in Financial Algebra/Mathematics will communicate effectively, using accurate mathematical language in a financial context. Students will interpret and analyze various functions, graphs and data in order to make responsible and wise financial decisions in the context of their personal lives regarding banking services, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed through opportunities for exploration and investigation of mathematical content and financial literacy topics:

<p>Mathematical Language in a Financial Context</p> <ul style="list-style-type: none"> • Demonstrate reasoning skills in developing, explaining, and justifying sound financial decisions. • Communicate effectively, using accurate mathematical language in a financial context. 	<p>The Algebra of Finance</p> <ul style="list-style-type: none"> • Apply algebraic skills and concepts to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.
<p>Financial Modeling with Functions</p> <ul style="list-style-type: none"> • Interpret and analyze various functions, graphs, and data to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions. 	<p>Modeling with Data</p> <ul style="list-style-type: none"> • Create, interpret, and evaluate financial models to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Applied Statistics:

<p>Mathematical Language in a Financial Context</p>	
<p>Communicate reasoning and decisions</p>	<p>Standards 1-3</p>
<p>Algebra/Mathematics of Finance</p>	
<p>Use algebraic reasoning and techniques</p>	<p>Standards 4-11</p>
<p>Financial Modeling with Functions</p>	
<p>Construct, graph, use, and interpret functions</p>	<p>Standards 12-27</p>
<p>Financial Modeling with Data</p>	
<p>Represent, summarize, and evaluate data</p>	<p>Standards 28-37</p>

Mathematical Language in a Financial Context

Cluster	Communicate reasoning and decisions.
M.FAM.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical decision making. (e.g., Demonstrate reasoning skills in creating and presenting a budget of monthly expenses based on a career pathway income, and analyze the soundness of the mathematical reasoning of others; determine outlook for a chosen career pathway and use the average salary to determine if the desired cost of living can be met.)
M.FAM.2	Communicate with and about mathematics in a financial context.
M.FAM.3	Communicate with and about mathematics in writing and orally, both independently and collaboratively, by preparing financial plans (e.g., Plan for an emergency savings fund that will last three to six months in the case of loss of income; determine the total percentage of income paid to taxes or the percentage of total salary that a benefits package represents).

Algebra/Mathematics of Finance

Cluster	Use algebraic reasoning and techniques.
M.FAM.4	Interpret parts of an expression or equation, such as terms, factors, and coefficients, in a variety of financial models including those found in stock markets, automobile financing and in banking contexts.
M.FAM.5	Create and solve linear equations and inequalities in one variable and use them to solve problems in financial applications that may include, but are not limited to stock markets, automobile ownership, business modeling, or employment. (e.g., Calculate wages by hourly rates or pay periods to make decisions about pay in a real world context.)
M.FAM.6	Create equations in two or more variables to represent relationships between quantities in a financial context; graph equations on coordinate axes with labels and scales. Financial contexts may include, but are not limited to stock markets, automobile ownership, business modeling employment, banking, consumer debt, and independent living decisions regarding taxes or planning for retirement. (e.g., Create a linear expense equation based on fixed and variable expenses and graph choosing an appropriate scale and origin for the graph)
M.FAM.7	Represent constraints in financial applications by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. (e.g., Create a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental; find the percentage of total salary that a benefits package represents; calculate taxes owed based on a given income and tax table and determine total percentage of income paid to taxes; calculate the gross pay and net pay using the FICA percentage (7.65%), retirement contribution, and worker's compensation insurance (employer match).)

M.FAM.8	Rearrange formulas for financial applications to highlight a quantity of interest, using the same reasoning as in solving equations. Know difference between growth and decay functions. (e.g., Solve the literal equation for exponential depreciation to find a depreciation rate and the literal equation for continuous interest to find the interest rate; apply the formula for average daily balance, $(\text{average daily balance} \times \text{APR} \times \text{days in billing cycle}) / 365$, using literal equations with varying APRs and billing cycles)).
M.FAM.9	Solve systems of linear equations exactly and approximately (e.g., with graphs) in making financial decisions, focusing on pairs of linear equations in two variables. (e.g., Create and solve a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental.)
M.FAM.10	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials that model financial situations. (e.g., Combine the polynomials that model income and expense to create a profit model.)
M.FAM.11	Solve quadratic equations in one variable in a financial context that may include, but are not limited to business modeling or employment decisions. (e.g., Given a quadratic equation that models a profit function, determine the break-even points; apply braking distance/stopping distance formulas to solve problems related to driving and safety data.)

Financial Modeling with Functions

Cluster	Construct, graph, use, and interpret functions.
M.FAM.12	Use functions to model financial situations. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$. (e.g., Develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; use linear and polynomial functions to evaluate and communicate quantities as required by Internal Revenue Service and Social Security Administration regulations and to determine when and why the models may be discontinuous.)
M.FAM.13	Use function notation in financial applications, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a financial context. (e.g., In making decisions regarding retirement income, apply the formula $A(t) = Pe^{rt}$ to determine future value.)
M.FAM.14	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Use this relationship in analyzing financial situations. (e.g., Compare the linear function modeling simple interest with the exponential function modeling compound interest.)

M.FAM.15	For a function that models a relationship between two quantities in financial contexts, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (e.g., Write, graph, and interpret the revenue (quadratic) function in comparison to the expense (linear) function using key features of the functions; reason quantitatively to compare subsidized and unsubsidized loans, as well as other forms of financial aid available to college students; calculate mortgage payments, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage.)
M.FAM.16	Interpret the parameters in a linear or exponential function in terms of a context. (e.g., Investigate and compare, using technology and regression, historical data to determine if automobile depreciation follows a linear or exponential model.)
M.FAM.17	Construct linear and exponential functions modeling financial contexts, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (e.g., Utilize linear and exponential functions to compare simple with compound interest.)
M.FAM.18	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.19	Calculate and interpret the average rate of change of a function modeling a financial context (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (e.g., Examine depreciation trends.)
M.FAM.20	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (e.g., Graph the linear, quadratic, or exponential curve that models the demand versus supply functions and find the equilibrium point with and without technology.)
M.FAM.21	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Utilize linear and exponential functions to compare simple with compound interest; calculate and compare using both the loan payment formula and payment schedules in table format, the monthly cost of purchasing an automobile, and discuss the feasibility of that payment in relation to monthly budget; compare two functions showing interest accrued when paying the minimum monthly payment over time compared to paying a larger monthly payment, and identify and compare the average rate of change between given time periods.)
M.FAM.22	Graph linear and quadratic functions and show intercepts, maxima, and minima. (e.g., In the model of a profit function, determine the break-even points, the maximum possible loss, and the maximum profit.)
M.FAM.23	Write a function that describes a relationship between two quantities in a financial context. (e.g., Calculate the costs associated with purchasing a vehicle, including leasing, purchasing with cash, or with a loan.)

M.FAM.24	Identify the effect on functions that model financial situations of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. (e.g., Identify the impact of a change in a constraint in a function that models retirement planning, business income and expenses, or employment benefits.)
M.FAM.25	Graph square root, cube root, and piecewise-defined functions that model financial situations, including step functions and absolute value functions. (e.g., Develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; analyze graphs of functions that model profit.)
M.FAM.26	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model financial situations, and translate between the two forms. (e.g., Create recursive and explicit models of sequences related to retirement planning; amortization schedules for a loan; comparing subsidized and unsubsidized loans, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage.)
M.FAM.27	Apply exponential formulas to solve for future and present value of investments by hand or with graphing technology. (e.g., $PV = FV \cdot (1/(1+r)^n)$ and $A(t) = Pe^{rt}$)

Financial Modeling with Data

Cluster	Represent, summarize, and evaluate data.
M.FAM.28	Represent data with plots on the real number line (dot plots, histograms, and box plots). Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.29	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit regression lines to scatterplots and make predictions based on lines of best fit. Find and interpret correlation coefficients of regression equations in financial situations. (e.g., Use scatter plots to show correlation between two funds, two stocks or even a stock and the general market or in business situations to forecast sales or to compare revenue to the number of units sold.)
M.FAM.30	Create a data display modeling financial situations. This may include, but is not limited to modeling the different savings options for a given investment at local banking establishments; calculating and comparing the monthly cost of purchasing an automobile using both the loan payment formula and payment schedules in table format; creating an amortization schedule through the use of spreadsheet technology and the formula tool for a loan given principle, term, monthly payment, and interest rate; creating representations of pay schedules using a variety of modeling technologies, and making decisions in a financial context based on those representations.

M.FAM.31	Summarize categorical data in various forms (e.g., two-way frequency tables, circle graphs, segmented bar charts). Interpret relative frequencies in the context of the data in making financial decisions.
M.FAM.32	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Data sets may address, but are not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.33	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.(e.g., Use units appropriately as a way to understand multi-step problems in relationship to understanding credit card fees and finance charges; compute distance, rate and time to solve problems to analyze driving and safety data, using single and multiple unit conversion; use and compare researched reaction times and vehicle velocity, as well as accepted equations to solve problems with braking distances.)
M.FAM.34	Use financial models from automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning to solve problems.
M.FAM.35	Evaluate reports based on data. Data may address, but is not limited to, planning for retirement or stock markets.
M.FAM.36	Use probability and expected value to analyze financial situations. (e.g., Model and compare automobile insurance policies.)
M.FAM.37	Evaluate the impact of taxes on business ownership including property tax, sales tax, social security, retirement, and disability benefits. Evaluate the impact of taxes on personal finance decisions.

