MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Statistics; Probability and Statistics

The Statistics course and the Probability and Statistics course are options for a one-credit year-long course or for a half-credit semester course.

The Statistics course is designed to be a full-year course that addresses all standards, including those identified by (+) sign, and that supplements the one-semester course with additional modeling experiences.

The Probability and Statistics course is designed to be a one-semester course and does not include the (+) standards.

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Knowledge of topics related to probability and statistics is critical to decision-making and to the analysis of data. The Statistics and the Probability and Statistics courses provide an opportunity to address the fundamental ideas and most commonly used techniques to organize and make sense of data. These courses build on knowledge of probability, randomness, and variability to provide students with an understanding of experimental design, estimation, hypothesis testing, and effective communication of experimental results. Using concepts of probability and statistics, students predict the likelihood of an event occurring, organize and evaluate data, and identify the significance of statements. Students investigate types of probability, determine probability and odds using multiple counting principles and distributions and apply the concepts to real-world problems. Statistical information collected and analyzed by students is used to investigate

ways of collecting, displaying, and analyzing data. Students differentiate, make predictions about and support the analysis of individual performance, characteristics of samples. They analyze and justify using statistical concepts to test validity of a hypothesis and of correlation as applied in real-world situations. Students identify a real life situation that involves statistical concepts, make a hypothesis as to the outcome; develop, justify, and implement a method to collect, organize and analyze data; generalize the results to make a conclusion, compare the hypothesis and the conclusion; present their findings using predictive and analytic tools in a clear and concise manner. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Descriptive Statistics	**Probability**
• Given a two-way table of relative frequencies that summarizes survey data relates a person's highest level of education and their role model, determine if a person whose highest level of education is a bachelor's degree is more likely to have a family member than a stranger as a role model.	• A cereal company is putting a prize in each box of cereal. The company is offering four different and evenly distributed prizes. How many boxes should one expect to need to buy to get all four prizes?
Probability Distributions	**Correlation and Regression**
• The heights of five women are measured to be 63 inches, 68 inches, 56 inches, 64 and 67 inches. Determine the expected value of the height of a randomly chosen woman.	• A random sample of beef hotdogs was taken and the amount of sodium (in mg) and calories were measured. Use the provided information to determine and use the regression equation to determine to find the amount of sodium in a beef hotdog that has 170 calories and in a beef hotdog that has 120 calories. Which of the calculated sodium levels is closer to the true sodium level? Why?

Confidence Intervals	Hypothesis Testing with One Variable
• Alyssa has over 500 songs saved on her phone. She wants to estimate the proportion of songs by a female artist. After taking a simple random sample of 50 songs, she finds that 20 of the sampled songs are by a female artist. Determine a 99% confidence interval for the proportion of songs on her phone that are by a female artist.	• Ellen has a pair of dogs and she noticed that they seem to breed more male puppies than female puppies. In the next litter of 12 puppies, there were 9 male puppies. Test the hypothesis that each puppy has an equal chance of 50% of being either male or female versus the alternative that the chance of a male puppy is greater. Look at the results of 1000 simulations, each simulating 12 puppies with a 50% chance of being male or female. According to the simulations, what is the probability of having 9 male puppies or more out of 12? If the observed outcome has a probability less than 1% under the tested hypotheses, reject the hypothesis. What should be concluded regarding the hypothesis?
Statistical Inference	
• A researcher wants to know if the children from three schools have equal mean IQ scores. Each school enrolls 1000 students. But there is neither the time of funding to test all 3000 students. Based on a simple random survey of 10 students from each school, perform and analyze an ANOVA test.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Descriptive Statistics	
Summarize, represent, and interpret data on single count or measurement variable.	Standards 1-8
Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 9-17
Probability Distributions	
Calculate expected values and use them to solve problems.	Standards 18-25
Correlation and Regression	
Interpret linear models.	Standards 26-29
Confidence Intervals	
Determine and interpret confidence intervals.	Standards 30-35
Hypothesis Testing with One Variable	
Use hypothesis testing in making and interpreting decisions.	Standards 36-44

Statistical Inference	
Determine and use correlation.	Standards 45-48
Use linear regression to predict and interpret.	Standards 49-53
Use statistical tests to determine a relationship.	Standards 54-59

Descriptive Statistics

Cluster	**Summarize, represent, and interpret data on single count or measurement variable.**
M.PS.1	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
M.PS.2	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
M.PS.3	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
M.PS.4	Evaluate reports based on data. Write a function that describes a relationship between two quantities.
M.PS.5	Represent data with plots on the real number line (dots plots, histograms, and box plots).
M.PS.6	Use statistics appropriate to the shape of the data distributions to compare center and spread of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.PS.7	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
M.PS.8	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.

Probability

Cluster	**Understand independence and conditional probability and use them to interpret data.**
M.PS.9	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
M.PS.10	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

M.PS.11	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. Instructional Note: Build on work with two-way tables from previous courses to develop understanding of conditional probability and independence.
M.PS.12	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. Instructional Note: Build on work with two-way tables from previous courses to develop understanding of conditional probability and independence.
M.PS.13	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
M.PS.14	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
M.PS.15	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
M.PS.16	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A) P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
M.PS.17	Use permutations and combinations to compute probabilities of compound events and solve problems.

Probability Distributions

Cluster	**Calculate expected values and use them to solve problems.**
M.PS.18	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
M.PS.19	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
M.PS.20	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated: find the expected value.
M.PS.21	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically to find the expected value.
M.PS.22	Weight the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values (e.g., find the expected payoff for a game of chance).
M.PS.24	Evaluate and compare strategies on the basis of expected values.
M.PS.25	Analyze decisions and strategies using probability concepts.

Correlation and Regression

Cluster	**Interpret linear models.**
M.PS.26	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.
M.PS.27	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in previous courses and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.PS.28	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.PS.29	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship is the focus.

Confidence Intervals

Cluster	**Determine and interpret confidence intervals.**
M.PS.30 (+)	Find the point estimate and margin of error in a given scenario.
M.PS.31 (+)	Construct and interpret confidence intervals for the population mean.
M.PS.32 (+)	Determine minimum sample size requirements when estimating mean, μ (population proportion).
M.PS.33 (+)	Interpret the t-distribution and use t-distribution table in real life scenarios.
M.PS.34 (+)	Construct confidence intervals when the sample size, n, is less than 30, population is normally distributed, and standard deviation, σ, is unknown.
M.PS.35 (+)	Interpret the chi-square distribution and use chi-square distribution table. Use the chi-square distribution to construct a confidence interval for the variance and standard deviation.

Hypothesis Testing with One Variable

Cluster	**Use hypothesis testing in making and interpreting decisions.**
M.PS.36 (+)	Interpret a hypothesis test; state a null hypothesis and an alternative hypothesis.
M.PS.37 (+)	Identify Type I and Type II errors and interpret the level of significance.
M.PS.38 (+)	Use one-tailed and two-tailed statistical tests to find p-value.
M.PS.39 (+)	Make and interpret decisions on comparing two hypotheses based on results of a statistical test. Write a claim for a hypothesis test.
M.PS.40 (+)	Find p values and test for mean and use in a z-test.
M.PS.41 (+)	Find critical values and rejection regions in a normal distribution. Use rejection regions for a z-test.
M.PS.42 (+)	Find critical values in a t-distribution and us the t-test to test a mean.
M.PS.43 (+)	Use the z-test to tests a population proportion, p.
M.PS.44 (+)	Find critical values for chi squared test. Use the chi squared test to test a variance or a standard deviation.

Statistical Inference

Cluster	**Determine and use correlation.**
M.PS.45 (+)	Find a correlation coefficient.
M.PS.46 (+)	Test a population correlation coefficient using a table.
M.PS.47 (+)	Perform a hypothesis test for a population correlation coefficient.
M.PS.48 (+)	Distinguish between correlation and causation.
Cluster	**Use linear regression to predict and interpret.**
M.PS.49 (+)	Find the equation of a regression line; predict y-values using a regression line.
M.PS.50 (+)	Interpret the types of variation about a regression line.
M.PS.51 (+)	Find and interpret the coefficient of determination.
M.PS.52 (+)	Find and interpret the standard error of estimate for a regression line; construct and interpret a prediction interval for y.
M.PS.53 (+)	Use technology to find a multiple regression equation, the standard error of estimate, and the coefficient of determination.

Cluster	Use statistical tests to determine a relationship.
M.PS.54 (+)	Use a contingency table to find expected frequencies.
M.PS.55 (+)	Use the chi-squared distribution to test whether a frequency distribution fits a claimed distribution and to test whether two variables are independent.
M.PS.56 (+)	Interpret the F-distribution and use an F-table to find critical values.
M.PS.57 (+)	Perform a two-sample F-test to compare two variances.
M.PS.58 (+)	Perform a two-sample F-test to compare two variances. Interpret the F-distribution and use an F-table to find critical values.
M.PS.59 (+)	Use one-way analysis of variance to test claims involving three or more means. Introduce two-way analysis of variance. Perform and analyze an ANOVA test.