West Virginia math4life

Educators' Guide for Mathematics

Grade 4

West Virginia DEPARTMENT OF EDUCATION

Grade Four

In grade four, students continue to build a strong foundation for higher mathematics. In previous grades, students develop place-value understandings, generalize written methods for addition and subtraction, and add and subtract fluently within 1000. They gain an understanding of single-digit multiplication and division and become fluent with such operations. They also develop an understanding of fractions built from unit fractions (adapted from Charles A. Dana Center 2012).

Mathematics Instruction

In grade four, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and Students also work toward fluency in addition and subtraction within 1,000,000 using the standard algorithm.

West Virginia College- and Career-Readiness Standards for Mathematics

The West Virginia College- and Career-Readiness Standards for Mathematics (WVBE Policy 2520.2B) emphasize key content, skills, and practices at each grade level and support three major principles:

- Instruction is focused on grade-level standards.
- Instruction should be attentive to learning across grades and to linking major topics within grades.
- Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade-level examples of these three major principles are indicated throughout this document.

Cluster headings can be viewed as the most effective way to communicate the focus and coherence of the standards. The instructional focus must be based on the depth of the ideas, the time needed to master those clusters, and their importance to future mathematics or the later demands of preparing for college and careers.

Teachers and administrators alike should note that the standards are not topics to be checked off after being covered in isolated units of instruction; rather, they provide content to be developed throughout the school year through rich instructional experiences presented in a coherent manner. West Virginia College- and Career-Readiness Standards for Mathematics are learning goals for students that must be mastered by the end of the fourth grade academic year in order for students to be prepared for the mathematics content at the fifth grade level.

Mathematical Fluency

Students demonstrate fluency of mathematical standards when they exhibit the following:

- Accuracy - ability to produce an accurate answer
- Efficiency - ability to choose an appropriate expedient strategy for a specific computation problem
- Flexibility - ability to use number relationships with ease in computation.

Connecting Mathematical Habits of Mind and Content

The Mathematical Habits of Mind **(MHM)** are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

The description of the Mathematical Habits of Mind is the same at all grade levels. However, the way these are utilized as students engage with and master new and more advanced mathematical ideas does change. The following chart presents examples of how the Mathematical Habits of Mind may be integrated into tasks appropriate for students in grade four.

Mathematical Habits of Mind—Explanation and Examples for Grade Four

Mathematical Habits of Mind	Explanation and Examples
MHM1 Make sense of problems and persevere in solving them.	In grade four, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students might use an equation strategy to solve a word problem. For example: "Chris bought clothes for school. She bought 3 shirts for \$12 each and a skirt for \$15. How much money did Chris spend on her new school clothes?" Students could solve this problem with the equation $3 \times \$12 + \$15 = a$. Students may use visual models to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often use another method to check their answers.
MHM2 Reason abstractly and quantitatively.	Grade-four students recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place-value concepts. Students might use array or area drawings to demonstrate and explain 154 × 6 as 154 added six times, and so they develop an understanding of the distributive property. For example: $154 \times 6 = (100 + 50 + 4) \times 6$ $= (100 \times 6) + (50 \times 6) + (4 \times 6)$ $= 600 + 300 + 24$ $= 924$ To reinforce students' reasoning and understanding, teachers might ask, "How do you know?" or "What is the relationship of the quantities?"

Mathematical Habits of Mind	Explanation and Examples
MHM3 Construct viable arguments and critique the reasoning of others.	Students may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions such as “How did you get that?”, “Explain your thinking.” and “Why is that true?” They not only explain their own thinking, but also listen to others’ explanations and ask questions. Students explain and defend their answers and solution strategies as they answer questions that require an explanation.
MHM4 Model with mathematics.	Students experiment with representing problem situations in multiple ways, including writing numbers; using words (mathematical language); creating math drawings; using objects; making a chart, list, or graph; and creating equations. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Students should be encouraged to answer questions such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?” Grade-four students evaluate their results in the context of the situation and reflect on whether the results make sense. For example, a student may use an area/array rectangle model to solve the following problem by extending from multiplication to division: “A fourth-grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?”
MHM5 Use appropriate tools strategically.	Students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they might use graph paper, a number line, or drawings of dimes and pennies to represent and compare decimals, or they might use protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. Students should be encouraged to answer questions such as, “Why was it helpful to use ________?”
MHM6 Attend to precision.	As grade-four students develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

Mathematical Habits of Mind	Explanation and Examples
MHM7 Look for and make use of structure.	Students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They generate number or shape patterns that follow a given rule. Teachers might ask, "What do you notice when ________?" or "How do you know if something is a pattern?"
MHM8 Look for and express regularity in repeated reasoning.	In grade four, students notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. They examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions. Students should be encouraged to answer questions such as "What is happening in this situation?" or "What predictions or generalizations can this pattern support?"

Adapted from Arizona Department of Education (ADE) 2010 and North Carolina Department of Public Instruction (NCDPI) 2013b.

Standards-Based Learning at Grade Four

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills from previous grades and provides exemplars to explain the content standards, highlight connections to the various Mathematical Habits of Mind (**MHM**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application.

Domain: Operations and Algebraic Thinking

In grade three, students focus on concepts, skills, and problem solving with single-digit multiplication and division (within 100). A critical area of instruction in grade four is developing understanding and fluency with multi-digit multiplication and developing understanding of division to find quotients involving multi-digit dividends.

Operations and Algebraic Thinking
Use the four operations with whole numbers to solve problems. **M.4.1** Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations. **M.4.2** Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem) and distinguish multiplicative comparison from additive comparison. **M.4.3** Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

In earlier grades, students focus on addition and subtraction of positive whole numbers and work with additive comparison problems (e.g., what *amount* would be added to one quantity in order to result in the other?). In grade four, students compare quantities multiplicatively for the first time.

In a multiplicative comparison problem, the underlying structure is that a *factor* multiplies one quantity to result in another quantity (e.g., b is n times as much as a, represented by $b = n \times a$. Students interpret a multiplication equation as a comparison and solve word problems involving multiplicative comparison (**M.4.1–M.4.2**) and should be able to identify and verbalize all three quantities involved: which quantity is being multiplied, which number tells how many times, and which number is the product. Teachers should be aware that students often have difficulty understanding the order and meaning of numbers in multiplicative comparison problems, and therefore special attention should be paid to understanding these types of problem situations (**MHM1**).

Example: Multiplicative Comparison Problems M.4.2

Unknown Product:
"Sally is 5 years old. Her mother is 8 times as old as Sally. How old is Sally's mother?" This problem takes the form $a \times b = ?$, where the factors are known but the product is unknown.

Unknown Factor (Group Size Unknown):
"Sally's mother is 40 years old. That is 8 times as old as Sally. How old is Sally?" This problem takes the form $a \times ? = p$, where the product is known, but the quantity being multiplied is unknown.

Unknown Factor 2 (Number of Groups Unknown):
"Sally's mother is 40 years old. Sally is 5 years old. How many times older than Sally is her mother?" This problem takes the form $? \times b = p$, where the product is known but the multiplicative factor is unknown.

Adapted from Kansas Association of Teachers of Mathematics (KATM) 2012, 4th Grade Flipbook.

In grade four, students solve various types of multiplication and division problems, which are summarized in the following table.

Types of Multiplication and Division Problems (Grade Four)

	Unknown Product	Group Size Unknown[1]	Number of Groups Unknown[2]
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 6 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there altogether? **Measurement example** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally and packed inside 3 bags, then how many plums will be in each bag? **Measurement example** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed, with 6 plums to a bag, then how many bags are needed? **Measurement example** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example** What is the area of a rectangle that measures 3 centimeters by 6 centimeters?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example** A rectangle has an area of 18 square centimeters. If one side is 3 centimeters long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example** A rectangle has an area of 18 square centimeters. If one side is 6 centimeters long, how long is a side next to it?

	Unknown Product	Group Size Unknown[1]	Number of Groups Unknown[2]
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 6 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? **Measurement example** A rubber band is 6 centimeters long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is three times as much as a blue hat costs. How much does a blue hat cost? **Measurement example** A rubber band is stretched to be 18 centimeters long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? **Measurement example** A rubber band was 6 centimeters long at first. Now it is stretched to be 18 centimeters long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

1. These problems ask the question, "How many in each group?" The problem type is an example of partitive or fair-share division.

2. These problems ask the question, "How many groups?" The problem type is an example of quotitive or measurement division.

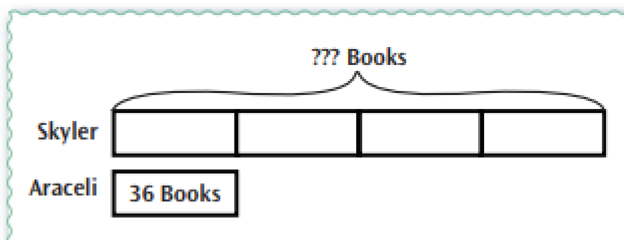
Students need many opportunities to solve contextual problems. A tape diagram or bar diagram can help students visualize and solve multiplication and division word problems. Tape diagrams are useful for connecting what is happening in the problem with an equation that represents the problem (**MHM2, MHM4, MHM5, and MHM7**).

Examples: Using Tape Diagrams to Represent Multiplication "Compare" Problems M.4.2

Unknown Product: "Skyler has 4 times as many books as Araceli. If Araceli has 36 books, how many books does Skyler have?"

Solution:

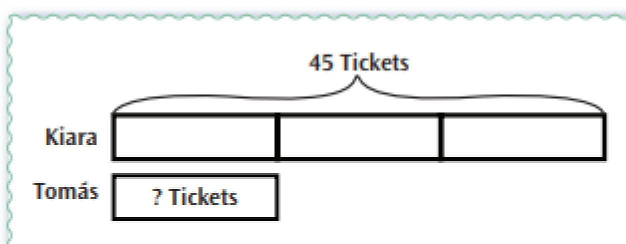
If we represent the number of books that Araceli has with a piece of tape or a bar, then the number of books Skyler has is represented by 4 pieces of tape or bars of the same size. Students can represent this as $4 \times 36 = \square$.



Unknown Factor (Group Size Unknown): "Kiara sold 45 tickets to the school play, which is 3 times as many as the number of tickets sold by Tomás. How many tickets did Tomás sell?"

Solution:

The number of tickets Kiara sold (the *product*) is known and is represented by 3 pieces of tape. The number of tickets Tomás sold would be represented by one same size piece of tape. This representation helps students see that the equations $3 \times \square = 45$ or $45 \div 3 = \square$ represent the problem.

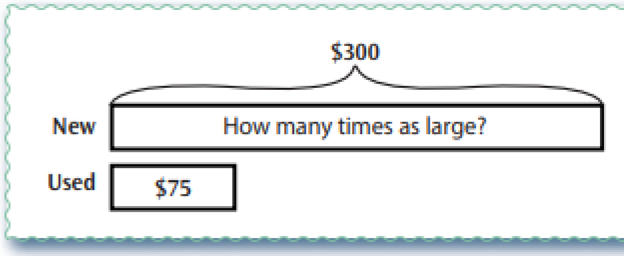


Examples: Using Tape Diagrams to Represent Multiplication "Compare" Problems M.4.2

Unknown Factor (Number of Groups Unknown): "A used bicycle costs \$75; a new one costs \$300. How many times as much does the new bike cost compared with the used bike?"

Solution:

The student represents the cost of the used bike with a piece of tape and decides how many pieces of this tape will make up the cost of the new bike. The representation leads to the equations $\square \times 75 = 300$ and $300 \div 75 = \square$.



Adapted from KATM 2012, 4th Grade Flipbook.

Additionally, students solve multi-step word problems using the four operations, including problems in which remainders must be interpreted (**M.4.3**). Students use estimation to assess the reasonableness of answers. They determine the level of accuracy needed to estimate the answer to a problem and select the appropriate method of estimation. This strategy gives rounding usefulness, instead of making it a separate topic that is covered arbitrarily.

Examples: Multi-Step Word Problems and Strategies Called for in Standard M.4.3

1. There are 146 students going on a field trip. If each bus holds 30 students, how many buses are needed?
   Solution:
   "Since $15 \div 30 = 5$, it seems like there should be around 5 buses. When we try to divide 146 by 30, we get 4 groups with 26 left over. This means that $146 = 4 \times 30 + 26$. There are 4 buses filled with 30 students, with a fifth bus holding only 26 students." (Given the context of the problem, one more than the quotient is the answer.)

2. Suppose that 250 colored pencils were distributed equally among 33 students for a geometry project. What is the largest number of colored pencils each student can receive?
   Solution:
   "Since $240 \div 30 = 8$, it seems like each student should receive close to 8 colored pencils. When we divide 250 by 33, we get 7 with a remainder of 19. This means that $250 = 33 \times 7 + 19$. This tells us that each student can have 7 colored pencils with 19 left over for the teacher."

3. Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs, with 6 bottles of water in each pack. Sarah wheels in 6 packs, each containing 6 bottles of water. About how many bottles of water still need to be collected?
   Solution:
   "First, I multiplied 3 packs by 6 bottles per pack, which equals 18 bottles. Then I multiplied 6 packs by 6 bottles per pack, which is 36 bottles. I added 18 and 36 and got 54. Then I subtracted $300 - 54$ and got 246. I know 18 is close to 20, and 20 plus 36 is around 50. Since we're trying to get to 300, we'll need about 250 more bottles, so my answer of 246 seems reasonable."

Adapted from KATM 2012, 4th Grade Flipbook.

As students compute and interpret multi-step problems with remainders (**M.4.3**), they also reinforce important mathematical practices as they make sense of the problem and reason about how the context is connected to the four operations (**MHM1, MHM2**).

Common Misconceptions

- Teachers may try to help their students by telling them that multiplying two numbers in a multiplicative comparison situation always makes the product bigger. While this is true with whole numbers greater than 1, it is <u>*not true*</u> when one of the factors is a fraction smaller than 1 (or when one of the factors is negative), something students will encounter in later grades. Teachers should be careful to emphasize that multiplying by a number greater than 1 results in a product larger than the original number (**M.4.1–2**).

- Students might be confused by the difference between 6 *more than* a number (additive) and 6 times a number (multiplicative). For example, using 18 and 6, a question could be "How much more is 18 than 6?" Thinking multiplicatively, the answer is 3; however, thinking additively, the answer is 12 (adapted from KATM 2012, 4th Grade Flipbook).

At grade four, students find all factor pairs for whole numbers in the range 1–100 (**M.4.4**). Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in grade six.

Operations and Algebraic Thinking
Gain familiarity with factors and multiples.
M.4.4 Find all factor pairs for a whole number in the range 1–100, recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Students extend the idea of decomposition to multiplication and learn to use the term *multiple*. Any whole number is a multiple of each of its factors. For example, 21 is a multiple of 3 and a multiple of 7 because 21 = 3 × 7. A number can be multiplicatively decomposed into equal groups (i.e., 3 equal groups of seven) and expressed as a product of these two factors (called *factor pairs*). The only factors for a *prime number* are 1 and the number itself. A *composite number* has two or more factor pairs. The number **1 is neither prime nor composite**. To find all factor pairs for a given number, students need to search systematically—by checking if 2 is a factor, then 3, then 4, and so on, until they start to see a "reversal" in the pairs. For example, after finding the pair 6 and 9 for 54, students will next find the reverse pair, 9 and 6.

Note: One fourth of all the numbers from 1 to 100 are prime (25 prime numbers). Using a hundreds chart and factoring out (cross out) all multiples of 2, 3, 5, and 7 (other than the numerals 2, 3, 5, & 7 themselves) the remaining numbers are prime.

Common Misconceptions

- Students may think the number 1 is a prime number or that all prime numbers are odd numbers. (Counter example: 2 has only two factors—1 and 2—and is therefore prime.)
- When listing multiples of numbers, students may omit the number itself. Students should be reminded that the smallest multiple is the number itself. Multiples of 3: 3, 6, 9, 12, 15…
- Students may think larger numbers have more factors. (Counter example: 98 has six factors: 1, 2, 7, 14, 49, and 98; 36 has nine factors: 1, 2, 3, 4, 6, 9, 12, 18, and 36.)

Having students share all factor pairs and explain how they found them will help students avoid some of these misconceptions.

Adapted from KATM 2012, 4th Grade Flipbook.

Instructional Focus

The concepts and terms *prime* and *composite* are new at grade four. As students gain familiarity with factors and multiples (**M.4.4**), they also reinforce and support major work at the grade, such as multi-digit arithmetic in the cluster “Use place-value understanding and properties of operations to perform multi-digit arithmetic” (**M.4.9–11**) and fraction equivalence in the cluster “Extend understanding of fraction equivalence and ordering” (**M.4.12–13**).

Understanding patterns is fundamental to algebraic thinking. In grade four, students generate and analyze number and shape patterns that follow a given rule (**M.4.5**).

Operations and Algebraic Thinking

Generate and analyze patterns.

M.4.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. (e.g., Given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.)

Students begin by reasoning about patterns, connecting a rule for a given pattern with its sequence of numbers or shapes. A *pattern* is a sequence that repeats or evolves in a predictable process over and over. A *rule* dictates what that process will look like. Patterns that consist of repeated sequences of shapes or growing sequences of designs can be appropriate for the grade. For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and then reason about how the dots are organized in the design to determine the total number of dots in the 100th design (**MHM2, MHM4, MHM5, MHM7**) [adapted from UA Progressions Documents 2011a].

Illustrative Mathematics (2013a) offers two examples of problems that can help students understand patterns: “Double Plus One” and “Multiples of Nine” (*https://www.illustrativemathematics.org/4* [accessed November 5, 2014]).

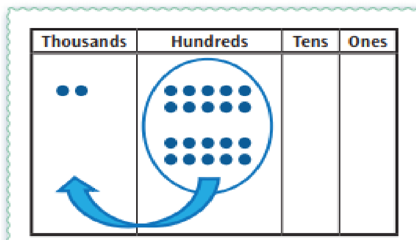
Instructional Focus
Numerical patterns (**M.4.5**) allow students to reinforce facts and develop fluency with operations. They also support major work in grade four in the cluster "Use place-value understanding and properties of operations to perform multi-digit arithmetic" (**M.4.9–11**).

Domain: Number and Operations in Base Ten

In grade four, students extend their work in the base-ten number system and generalize previous place-value understanding to multi-digit whole numbers (less than or equal to 1,000,000).

Number and Operations in Base Ten
Generalize place value understanding for multi-digit whole numbers.
M.4.6 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right (e.g., recognize that 700 ÷ 70 = 10 by applying concepts of place value and division).
M.4.7 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons.
M.4.8 Use place value understanding to round multi-digit whole numbers to any place.

Students read, write, and compare numbers based on the meaning of the digits in each place (**M.4.6–M.4.7**). In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. Students can come to see and understand that multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left (adapted from UA Progressions Documents 2012b). Students can develop their understanding of millions by using a place-value chart to understand the pattern of *times ten* in the base-ten system; for example, 20 hundreds can be bundled into 2 thousands.



2 thousands = 20 hundreds

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones
						4
					4	0
				4	0	0
			4	0	0	0
		4	0	0	0	0
	4	0	0	0	0	0
	4	4	4	4	4	4

"Four hundred forty-four thousand, four hundred forty-four"

Students need multiple opportunities to use real-world contexts to read and write multi-digit whole numbers. As they extend their understanding of numbers to 1,000,000, students reason about the magnitude of digits in a number and analyze the relationships of numbers. They can build larger numbers by using graph paper and labeling examples of each place with digits and words (e.g., *10,000* and *ten thousand*).

To read and write numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (e.g., *thousand, million*). Layered place-value cards such as those used in earlier grades can be put on a frame with the base-thousand units labeled below (**MHM2, MHM3, MHM8**).

Grade-four students build on the grade-three skill of rounding to the nearest 10 or 100 to round multi-digit numbers and to make reasonable estimates of numerical values (**M.4.8**).

Examples: Rounding Numbers in Context M.4.8 (MHM4)

The population of the fictional Midtown, USA, was last recorded as 76,398. The city council wants to round the population to the nearest thousand for a business brochure. What number should they round the population to?

Solution:

When numbers are stacked vertically, the relationships between the numbers may be more clearly identified. Students might think: "I know the answer is either 76,000 or 77,000. If I write 76,000 below 76,398 and 77,000 above it, I can see that the midpoint is 76,500, which is *above* 76,398. This tells me the population should round to 76,000."

77,000
76,398
76,000

Adapted from ADE 2010.

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

M.4.9

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

M.4.10

Multiply a whole number of up to four digits by a one-digit whole number, multiply two two-digit numbers, using strategies based on place value and the properties of operations and illustrate and explain the calculation by using equations, rectangular arrays and/or area models.

M.4.11

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.

At grade four, students become fluent with addition and subtraction with multi-digit whole numbers to 1,000,000 using standard algorithms (**M.4.9**). A central theme in multi-digit arithmetic is to encourage students to develop methods they understand and can explain rather than merely following a sequence of directions, rules, or procedures they do not understand. In previous grades, students built a conceptual understanding of addition and subtraction with whole numbers as they applied multiple methods to compute and solve problems. The emphasis in grade four is on the power of the regular one-for-ten trades between adjacent places that let students extend a method width which they are familiar. Since students in grades two and three have been using at least one method that will generalize to 1,000,000, this extension in grade four should transpire quickly.

Thus, allowing more time for students to expand their understanding of multiplication and division (**M.4.10–M.4.11**).

FLUENCY
West Virginia's College- and Career-Readiness Standards for Mathematics (K–6) set expectations for fluency in computations using the standard algorithm (e.g., "*Fluently* add and subtract multi-digit whole numbers using the standard algorithm" [**M.4.9**]). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. The word *fluent* is used in the standards to mean "reasonably fast and accurate" and possessing the ability to use certain facts and procedures with enough facility that using such knowledge does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of knowing some answers, utilizing patterns to obtain some answers, and employing other strategies to obtain some answers.

Adapted from UA Progressions Documents 2011a.

In grade four, students extend multiplication and division to include whole numbers greater than 100. Students should use methods they understand and can explain to multiply and divide. The standards (**M.4.10–M.4.11**) call for students to use visual representations such as area and array models that students draw and connect to equations, as well as written numerical work, to support student reasoning and explanation of methods. By reasoning repeatedly about the connections between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities.

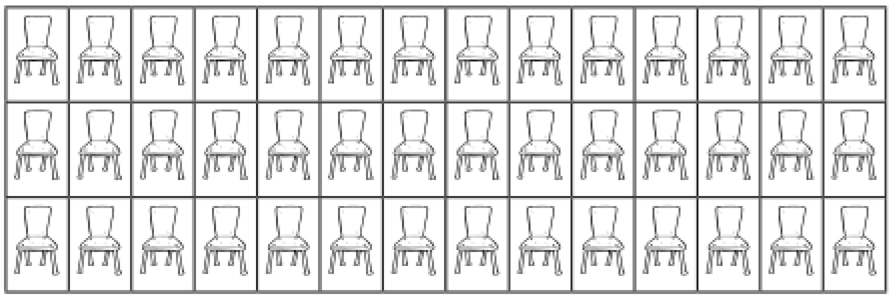
Students can use area models to represent various multiplication situations. The rows can represent the equal groups of objects in the situation, and students then imagine that the objects lie in the squares forming an array. With larger numbers, such array models become too difficult to draw, so students can make sketches of rectangles and then label the resulting product as the number of things or square units. When area models are used to represent an actual area situation, the two factors are expressed in length units (e.g., *cm*) while the product is in square units (e.g., cm^2).

Example: Area Models and Strategies for Multiplication with a Single-Digit Multiplier M.4.10

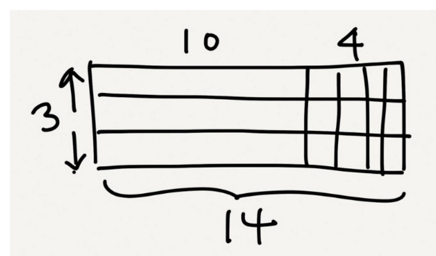
Chairs are being set up for a small play. There should be 3 rows of chairs and 14 chairs in each row. How many chairs will be needed?

Solution:

As in grade three, when students first make the connection between array models and the area model, students may start by drawing a sketch of the situation. They can then be reminded to see the chairs as if surrounded by unit squares and hence a model of a rectangular region. Students represent the problem with base-ten blocks or math drawings (**MHM2, MHM5**). This representation provides a visual reference for writing the expression 3 × (10 + 4).



Making a sketch like the one above becomes cumbersome, so students move toward representing such drawings more abstractly, with rectangles, as shown to the right. This builds on the work begun in grade three. Such diagrams help children see the distributive property: 3 × 14 can be written as 3 × (10 + 4). Then the multiplications may be performed separately and adding the resulting products provide the number of needed chairs: 3 × (10 + 4) = 3 × 10 + 3 × 4. The answer is (42 chairs).



In grade three, students multiply single-digit numbers by multiples of 10 (**M.4.8**). This idea is extended in grade four. For example, since, the following equations and statements must be true:

- 6 × 70 = 420, since this is "6 times 7 tens," which is 42 tens.
- 6 × 700 = 4200, since this is "6 times 7 hundreds," which is 42 hundreds.
- 6 × 7000 = 42,000, since this is "6 times 7 thousands," which is 42 thousands.
- 60 × 70 = 4200, since this is "60 times 7 tens," which is 420 tens, or 4200.

Math drawings and base-ten blocks support the development of these *extended multiplication facts*. The ability to find products such as these is important when variations of the standard algorithm are used for multi-digit multiplication, as described in the following examples.

Examples: Developing Written Methods for Multi-Digit Multiplication M.4.10

Find the product: 6 × 729

	700	**+ 20**	**+ 9**
6	6 × 700 = 6 groups of 7 hundreds = 42 hundreds = 4200	6 × 20 = 6 groups of 2 tens = 12 tens = 120	6 × 9 = 54

Solution:

Sufficient practice with drawing rectangles (or constructing them with base-ten blocks) will help students understand that the problem can be represented with a rectangle such as the one shown. The product is given by the total area: 6 × 729 = 6 × 700 + 6 × 20 + 6 × 9. Understanding extended multiplication facts allows students to find the *partial products* quickly. Students can record the multiplication in several ways:

Left to right, showing the partial products.

729	
× 6	Thinking:
4200	6 × 7 hundreds
120	6 × 2 tens
54	6 × 9
4374	

Right to left, showing the partial products.

729	
× 6	
54	6 × 9
120	6 × 2 tens
4200	6 × 7 hundreds
4374	

Examples: Developing Written Methods for Multi-Digit Multiplication M.4.10

Find the product: 27 × 65

Solution:
A rectangle is drawn, and like base-ten units (i.e., tens and ones) are represented by sub-regions of the rectangle. Repeated use of the distributive property shows that:

27 × 65 = (20 + 7) × 65 = 20 × 65 + 7 × 65
= 20 × (60 + 5) + 7 × (60 + 5)
= 20 × 60 + 20 × 5 + 7 × 60 + 7 × 5

	60	**+ 5**
20	20 × 60 = 2 tens times 6 tens = 12 hundreds = 1200	20 × 5 = 2 tens × 5 = 10 tens = 100
+ 7	7 × 60 = 7 × 6 tens = 42 tens = 420	7 × 5 = 35

The product is again given by the total area: 1200 + 100 + 420 + 35 = 1755

Below are two written methods for recording the steps of the multiplication.

Show the partial products

65	Thinking:
× 27	
35	7 × 5
420	7 × 6 tens
100	2 tens × 5
1200	2 tens × 6 tens
1755	

General methods for computing quotients of multi-digit numbers and one-digit numbers (**M.4.11**) rely on the same understandings as for multiplication, but these are cast in terms of division. For example, students may see division problems as knowing the area of a rectangle but not one side length (the quotient), or as finding the size of a group when the number of groups is known (measurement division).

Example: Using the Area Model to Develop Division Strategies M.4.11

Find the quotient: 750 ÷ 6

	? hundreds + ? tens + ? ones
6	750

Possible Student Solution: "Just as with multiplication, I can set-up a representation of this problem as a rectangle, but with one side unknown since this is the same as ? × 6 = 750. I find out what the number of hundreds would be for the unknown side length; that's 1 hundred or 100, since 100 × 6 = 600, and that's as large as I can go. Then, I have 750 - 600 = 150 square units left, so I find the number of tens that are in the other side. That's 2 tens, or 20, since 20 × 6 = 120. Last, there are 150 - 120 = 30 square units left, so the number of ones on the other side must be 5, since 5 x 6 = 30."

	100	**+ 20**	**+ 5**
6	750 - 600 150	150 - 120 30	30 - 30 0

One way students can record this is shown at right: *partial quotients* are stacked atop one another, with zeros included to indicate place value and as a reminder of how students obtained the numbers. The full quotient is the sum of these stacked numbers.

	5	
	20	125
	100	
6	750	
	- 600	
	150	
	- 120	
	30	
	- 30	
	0	

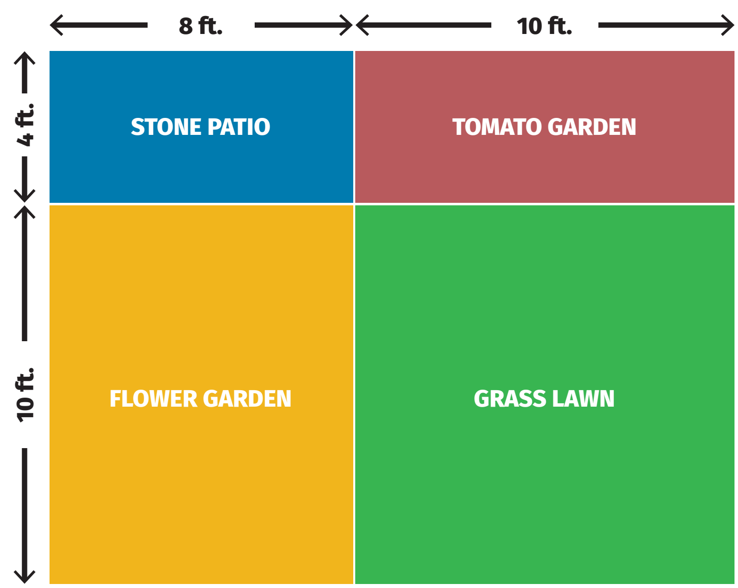
General methods for multi-digit division computation include decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this method relies on the distributive property. This work continues in grade five and culminates in fluency with the standard algorithm in grade six.

In grade four, students also find whole-number quotients with remainders (**M.4.11**) and learn the appropriate way to write the result. For instance, students divide and find that 195 ÷ 9 = 21, with 6 left over. This can be written as 195 = 21(9) + 6. When put into a context, the latter equation makes sense. For instance, if 195 books are distributed equally among 9 classrooms, then each classroom gets 21 books, and 6 books will be left over. The equation 195 = 9(21) + 6 is closely related to the equation 195 ÷ 9 = $21\frac{6}{9}$ which students will write in later grades. The remainder should be interpreted in context of the problem.

As students decompose numbers to solve division problems, they also reinforce important mathematical practices such as seeing and making use of structure (**MHM7**). As they illustrate and explain calculations, they model (**MHM4**), strategically use appropriate drawings as tools (**MHM5**), and attend to precision (**MHM6**) using base-ten units.

The following table presents a sample classroom activity that connects the content standards to the Mathematical Habits of Mind.

Connecting to the Mathematical Habits of Mind --Grade Four

Standards Addressed	Explanation and Examples
Connections to Mathematical Habits of Mind **MHM1** Students make sense of the problem when they see that the measurements on the side and top of the diagram persist and yield the measurements of the smaller areas. **MHM2** Students reason abstractly as they represent the areas of the yard as multiplication problems to be solved. **MHM5** Students use appropriate tools strategically when they apply the formula for the area of a rectangle to solve the problem. They organize their work in a way that makes sense to them.	**Sample Problem:** What are the areas of the four sections of Mr. Griffin's backyard? The yard has a stone patio, a tomato garden, a flower garden, and a grass lawn. What is the area of his entire backyard? How did you find your answer? 

MHM7 Teachers can use this problem and similar problems to illustrate the distributive property of multiplication. In this case, we find that 18 × 14 = (10 × 14) + (8 × 14) = (10 × 10) + (10 × 4) + (8 × 10) + (8 × 4).	18 × 14 Area of Stone Patio → 32 (4×8) Area of Tomato Garden → 40 (4×10) Area of Flower Garden → 80 (10×8) Area of Grass Lawn → 100 (10×10) Area of Entire Backyard → 252 Square feet
Content Standards **M.4.10** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and properties of operations. Illustrate and explain the calculation using equations, rectangular arrays, and/or area models. **M.4.21** Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	**Solution.** The areas of the four selections are 32 square feet, 40 square feet, 80 square feet, and 100 square feet, respectively. The area of the entire backyard is the sum of these areas: (32 + 40 + 80 + 100), or 252 square feet. This is the same as finding the product of 18 × 14 = 252 square feet. **Classroom Connections.** The purpose of this task is to illuminate the connection between the area of a rectangle as representing the product of two numbers and the partial products algorithm for multiplying multi-digit numbers. In this algorithm, which is shown beneath the area model, each digit of one number is multiplied by each digit of the other number, and the "partial products" are written down. The sum of these partial products is the product of the original numbers. Place value can be emphasized by specifically reminding students that if we multiply the 2 tens together, since each represents 1 ten, the product is 100. Finally, the area model provides a visual justification for how the algorithm works.

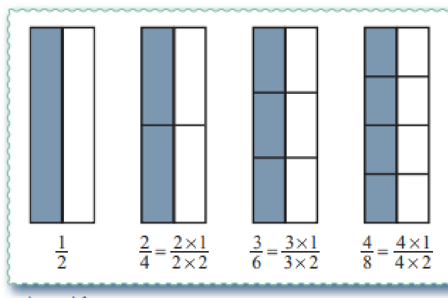
Domain: Number and Operations–Fractions

Student proficiency with fractions is essential to success in algebra. In grade three, students develop an understanding of fractions as built from unit fractions. A critical area of instruction in grade four is fractions, including developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers. In grade four, fractions include those with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Number and Operations–Fractions
Extend understanding of fraction equivalence and ordering. **M.4.12** Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **M.4.13** Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, = or <, and justify the conclusions by using a visual fraction model.

Grade-four students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction (e.g., $\frac{a}{b} = \frac{n \times a}{n \times b}$, for $n \neq 0$). Students use visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size (**M.4.12**). This property forms the basis for much of the work with fractions in grade four, including comparing, adding, and subtracting fractions and the introduction of finite decimals.

Students use visual models to reason about and explain why fractions are equivalent. For example, the area models to the right represent fractions equivalent to $\frac{1}{2}$. Students in grade three visually justify that all the models represent the same amount. Fourth-grade students reason about *why* it is true that $\frac{1}{2} = \frac{2 \times 1}{2 \times 2} = \frac{3 \times 1}{3 \times 2} = \frac{4 \times 1}{4 \times 2}$, and so on.



Adapted from ADE 2010.

Students use reasoning such as the following: when a horizontal line is drawn through the center of the first model to obtain the second, both the number of equal parts and the number of those parts being counted are doubled (2 × 2 = 4 in the denominator, 2 × 1 = 2 in the numerator, respectively). Even though there are more parts counted, the parts are smaller. Students make connections between the models and the fractions represented by the models in the way both the parts and wholes are counted. They begin to generate a rule for writing equivalent fractions. Students also emphasize the inversely related changes: the number of unit fractions becomes larger, but the size of the unit fraction becomes smaller.

Students should have repeated opportunities to use math drawings such as these (and the ones that follow in this document) to understand the general method for finding equivalent fractions. Of course, students may also realize that the rule works both ways. For example:

$$\frac{28}{35} = \frac{7 \times 4}{7 \times 5} = \frac{4}{5}$$

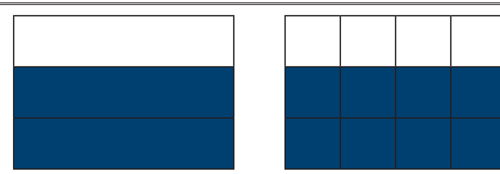
Teachers must be careful to avoid overemphasizing this "simplifying" of fractions, **as there is no mathematical reason for doing so**—although, depending on the problem context, one form (renamed or not renamed) may be more desirable than another. **In particular, teachers should avoid using the term *reducing* fractions for this process, as the value of the fraction itself is *not* being reduced.** A more neutral term, such as ***renaming*** (which hints at these fractions being different names for the same amount), or simplifying allows teachers to refer to this strategy with less potential for student misunderstanding.

Instructional Focus
It is true that one can justify that $\frac{a}{b} = \frac{n \times a}{n \times b}$ by arguing as follows: $$\frac{n \times a}{n \times b} = \frac{n}{n} \times \frac{a}{b} = 1 \times \frac{a}{b} = \frac{a}{b}$$ That is, we are simply multiplying by 1 in the form of , since students have not yet encountered the general notion of fraction multiplication in grade four. However, this argument should be avoided in favor of developing an understanding with diagrams and reasoning about the size and number of parts that are created in this process. In grade five, students will learn the general rule that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

Examples: Reasoning with Diagrams That $\frac{a}{b} = \frac{n \times a}{n \times b}$ M.4.12

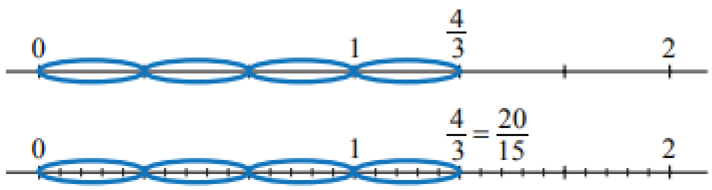
Using an Area Model.

The area of the rectangle represents one whole. In the illustrations provided, the rectangle on the left shows the area divided into three rectangles of equal area (thirds), with two of them shaded (2 pieces of size $\frac{1}{3}$), representing $\frac{2}{3}$.



In the figure on the right, the vertical lines divide the parts (the thirds) into smaller parts. There are now 4 × 3 smaller rectangles of equal area, and the shaded area now comprises 4 × 2 of them, so it represents $\frac{4 \times 2}{4 \times 3} = \frac{8}{12}$.

Using a Number Line.
The first number line shown below indicates $\frac{4}{3}$. Each unit length is divided into three equal parts. When each $\frac{1}{3}$ is further divided into 5 equal parts, (as indicated in the second number line shown below) there are now 5 × 3 of these new equal parts. Since 4 of the $\frac{1}{3}$ parts were circled before, and each of these has been subdivided into 5 parts, there are now 5 × 4 of these new small parts. Therefore, $\frac{4}{3} = \frac{5\times4}{5\times3} = \frac{20}{15}$.



Adapted from UA Progressions Documents 2013a.

Creating equivalent fractions by dividing and shading squares or circles and matching each fraction to its location on the number line can reinforce students' understanding of fractions.

Students apply their new understanding of equivalent fractions to compare two fractions with different numerators and different denominators (**M.4.13**). They compare fractions by using benchmark fractions and finding common denominators or common numerators. Students explain their reasoning and record their results using the symbols >, =, and <.

Examples: Comparing Fractions M.4.13

1. Students compare fractions to benchmark fractions — for example, comparing to $\frac{1}{2}$ when comparing $\frac{3}{8}$ and $\frac{2}{3}$. Students see that $\frac{3}{8} < \frac{4}{8}$ $\left(\frac{4}{8} = \frac{1}{2}\right)$, and that since $\frac{2}{3} = \frac{4}{6}$ and $\frac{4}{6} > \frac{3}{6}$ $\left(\frac{3}{6} = \frac{1}{2}\right)$, it must be true that $\frac{3}{8} < \frac{2}{3}$.

2. Students compare $\frac{5}{8}$ and $\frac{7}{12}$ by writing them with a common denominator. They find that $\frac{5}{8} = \frac{5\times12}{8\times12} = \frac{60}{96}$ and $\frac{7}{12} = \frac{7\times8}{12\times8} = \frac{56}{96}$ and reason therefore that $\frac{5}{8} > \frac{7}{12}$. **Notice that students do not need to find the smallest common denominator for two fractions; any common denominator will work.**

3. Students can also find a common numerator to compare $\frac{5}{8}$ and $\frac{7}{12}$. They find that $\frac{5}{8} = \frac{5\times7}{8\times7} = \frac{35}{56}$ and $\frac{7}{12} = \frac{7\times5}{12\times5} = \frac{35}{60}$. Then they reason that, since parts of size $\frac{1}{56}$ are larger than parts of size $\frac{1}{60}$ when the whole is the same, $\frac{5}{8} > \frac{7}{12}$.

Adapted from ADE 2010.

In grade four, students extend previous understanding of addition and subtraction of whole numbers to add and subtract fractions with like denominators (**M.4.14**).

Number and Operations–Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

M.4.14

Understand the fraction a/b, with a > 1, as the sum of a of the fractions 1/b.

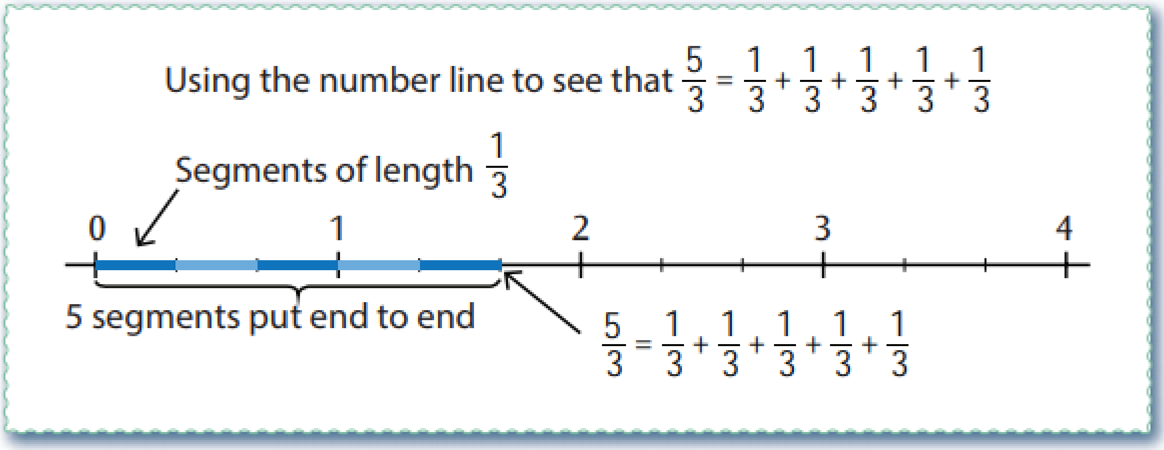
a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation and justify decompositions by using a visual fraction model (e.g., 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8).
c. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators by using visual fraction models and equations to represent the problem.

Students begin building fractions from unit fractions by understanding a fraction, $\frac{a}{b}$, as a sum of the unit fractions, $\frac{1}{b}$. In grade three, students learned that the fraction $\frac{a}{b}$ represents *a* parts when a whole is broken into *b* equal parts (i.e., parts of size $\frac{1}{b}$.) However, in grade four, students connect this understanding of a fraction with the operation of addition; for instance, they see now that if a whole is broken into 4 equal parts and 5 of them are taken, then this is represented by both $\frac{5}{4}$ and the expression $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ (**M.4.14b**). They experience composing fractions from and decomposing fractions into sums of unit fractions and non-unit fractions in this general way—for example, by seeing $\frac{5}{4}$ also as:

$$\boxed{\frac{1}{4}+\frac{1}{4}+\frac{3}{4}} \text{ or } \boxed{\frac{2}{4}+\frac{3}{4}} \text{ or } \boxed{\frac{1}{4}+\frac{3}{4}+\frac{1}{4}}$$

Students write and use unit fractions while working with standard **M.4.14b,** which supports their conceptual understanding of adding fractions and solving problems (**M.4.14a**, **M.4.14d**). Students write and use unit fractions while decomposing fractions in several ways (**M.4.14b**). This work helps students understand addition and subtraction of fractions (**M.4.14a**) and how to solve word problems involving fractions with the same denominator (**M.4.14d**). Writing and using unit fractions also helps students avoid the common misconception of adding two fractions by adding their numerators and adding the denominators — for example, *erroneously* writing $\frac{1}{2}+\frac{5}{6}=\frac{6}{8}$. In general, the meaning of addition is the same for both fractions and whole numbers. Students understand addition as "putting together" *like units*, and they visualize how fractions are *built from unit fractions* and that a fraction is a *sum of unit fractions*.

Students may use visual models to support this understanding—for example, showing that $\frac{5}{3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ by using a number line model (**MHM1, MHM2, MHM4, MHM6, and MHM7**).



Source: UA Progressions Documents 2013a.

Students add or subtract fractions with like denominators, including *mixed numbers* (**M.4.14a, c**), and solve word problems involving fractions (**M.4.14d**). They use their understanding that every fraction is composed of unit fractions to make connections such as this:

$$\frac{7}{5}+\frac{4}{5}=\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}\ +\ \frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{7+4}{5}$$

This allows students to develop a general principle that $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$. Using similar reasoning, students understand that $\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$.

Students also compute sums of whole numbers and fractions, realizing that any whole number can be written as an equivalent number of unit fractions of a given size. For example, they find the sum $3+\frac{7}{2}$ in the following way:

$$3+\frac{7}{2}=\frac{6}{2}+\frac{7}{2}=\frac{13}{2}$$

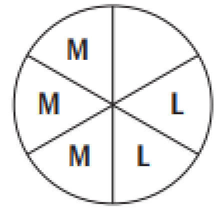
Understanding this method of adding a whole number and a fraction allows students to accurately convert mixed numbers into fractions, as in this example:

$$4\frac{5}{8}=4+\frac{5}{8}=\frac{32}{8}+\frac{5}{8}=\frac{37}{8}$$

Students should develop a solid understanding that a *mixed number* indicates the sum of a whole number and a fraction (i.e., $a\frac{b}{c}=a+\frac{b}{c}$). They should also learn a method for converting mixed numbers to fractions that is connected to the meaning of fractions (such as the one demonstrated above), rather than typical rote methods.

Examples: Reasoning with Addition and Subtraction of Fractions M.4.14

1. Mary and Lacey share a pizza. Mary ate $\frac{3}{6}$ of the pizza and Lacey ate $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat altogether? (**MHM3**, **MHM4**). Use the picture of a pizza to explain your answer.

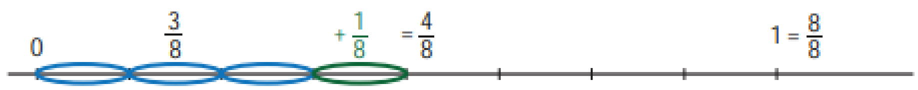


Solution:
"I labeled three of the one-sixth pieces for Mary and two of the one-sixth pieces for Lacey. I can see that altogether, they've eaten five of the one-sixth pieces, or $\frac{5}{6}$ of the pizza. Also, I know that $\frac{3}{6}+\frac{2}{6}=\frac{2+3}{6}=\frac{5}{6}$."

Adapted from KATM 2012, 4th Grade Flipbook.

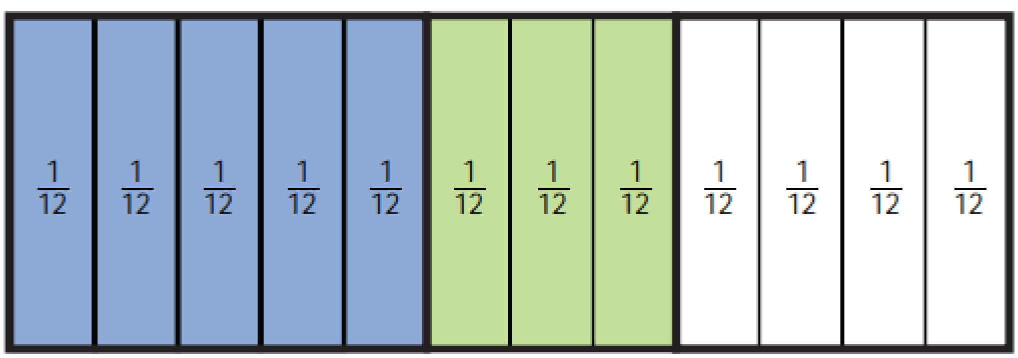
2. Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Is it enough to complete the packaging?

Solution:
"I know I need to find $5\frac{3}{8}+3\frac{1}{8}$ to find out how much they have altogether. I know that Susan and Maria have 3 + 5 = 8 feet of ribbon plus the other $\frac{3}{8}+\frac{1}{8}$ feet of ribbon. Altogether, this is $8\frac{4}{8}$ feet of ribbon, which means they have enough ribbon to do their packaging. They even have $\frac{1}{8}$ feet of ribbon left."



Adapted from KATM 2012, 4th Grade Flipbook.

3. Elena, Matthew, and Kevin painted a wall. Elena painted $\frac{5}{12}$ of the wall and Matthew painted $\frac{3}{12}$ of the wall. Kevin painted the rest. How much of the wall did Kevin paint? Use the picture below to help find your answer.



Solution:
"By shading what Elena and Matthew painted, I can show in the picture that Elena and Matthew painted $\frac{8}{12}$ of the wall. The remaining part that Kevin painted $\frac{4}{12}$ was of the wall. I can write this as $\frac{12}{12}-\frac{8}{12}=\frac{4}{12}$, or $1-\frac{8}{12}=\frac{4}{12}$, or $1-\frac{5}{12}-\frac{3}{12}=\frac{4}{12}$."

Adapted from New York State Education Department (NYSED) 2012.

Number and Operations–Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

M.4.15

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b, (e.g., use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)).

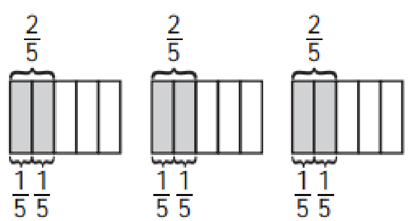
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number (e.g., use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. In general, n × (a/b) = (n × a)/b).

c. Solve word problems involving multiplication of a fraction by a whole number by using visual fraction models and equations to represent the problem (e.g., If each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?).

In grade three, students learned that 3 × 7 can be represented as the total number of objects in 3 groups of 7 objects and that they could solve this by adding 7 + 7 + 7. Fourth-grade students apply this concept to fractions, understanding a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ (**M.4.14**). This understanding is connected with standard **M.4.15,** and students make the shift to $\frac{5}{3}$ see as $5 \times \frac{1}{3}$. For example, they see the following:

$$\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 5 \times \frac{1}{3}$$

Students extend this understanding to make meaning of the product of a whole number and a fraction (**M.4.15b**)—for example, seeing $3 \times \frac{2}{5}$ in the following ways:



$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}$$

Source: UA Progressions Documents 2013a.

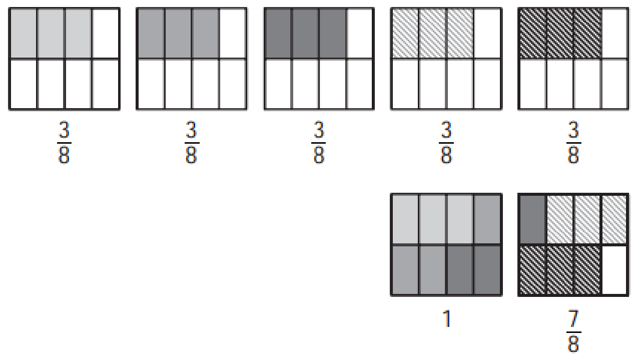
Additionally, students are presented with opportunities to work with word problems involving multiplication of a fraction by a whole number to relate situations, models, and corresponding equations (**M.4.15c**).

Example: Multiplying a Fraction by a Whole Number M.4.15c

Each person at a dinner party eats $\frac{3}{8}$ of a pound of pasta. There are 5 people at the party. How many pounds of pasta are needed? Pasta comes in 1-pound boxes. How many boxes of pasta should be bought? (**MHM1, MHM2, MHM7**)

***Solution*:**

If 5 rectangles are drawn, with $\frac{3}{8}$ of a pound shaded in each rectangle, then students see that they are computing $5 \times \frac{3}{8} = \frac{15}{8}$.



The separate eighths can be collected together to illustrate that a total of $1\frac{7}{8}$ pounds of pasta will be needed for the party. This means that 2 boxes of pasta should be bought.

Adapted from ADE 2010 and NCDPI 2013b.

Number and Operations–Fractions

Understand decimal notation for fractions, and compare decimal fractions.

M.4.16

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100 (e.g., express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100).[3] Instructional Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

M.4.17

Use decimal notation for fractions with denominators 10 or 100 (e.g., rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram).

M.4.18

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the **same whole**. Record the results of comparisons with the symbols >, = or <, and justify the conclusions by using a visual model.

3. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

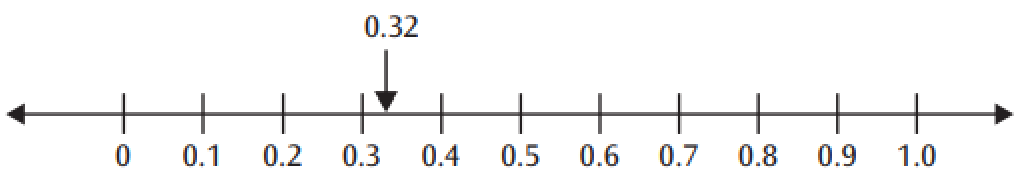
Grade-four students develop an understanding of decimal notation for fractions and compare *decimal fractions* (fractions with a denominator of 10 or 100). This work lays the foundation for performing operations with decimal numbers in grade five. Students learn to add decimal fractions by converting them to fractions with the same denominator (**M.4.16**). For example, students express $\frac{3}{10}$ as $\frac{30}{100}$ before they add $\frac{30}{100} + \frac{4}{100} = \frac{34}{100}$. Students can use graph paper, base-ten blocks, and other place-value models to explore the relationship between fractions with denominators of 10 and 100 (adapted from UA Progressions Documents 2013a).

In grade four, students first use decimal notation for fractions with denominators 10 or 100 (**M.4.17**), understanding that the number of digits to the right of the decimal point indicates the number of zeros in the denominator. Students make connections between fractions with denominators of 10 and 100 and place value. They read and write decimal fractions; for example, students say 0.32 as "thirty-two hundredths" and learn to flexibly write this as both 0.32 and $\frac{32}{100}$.

Instructional Focus

To reinforce student understanding, teachers are urged to consistently use place-value-based language when naming decimals—for example, by saying "four-tenths" rather than "point four" when referring to 0.4, and by saying "sixty-eight hundredths" as opposed to "point sixty eight" or "point six eight" when referring to 0.68.

Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. They reason that $\frac{32}{100}$ is a little more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$(or $\frac{4}{10}$). It is closer to $\frac{30}{100}$, so it would be placed on the number line near that value (**MHM2, MHM4, MHM5, MHM7**).



Students compare two decimals to hundredths by reasoning about their size (**M.4.18**).They relate their understanding of the place-value system for whole numbers to fractional parts represented as decimals. Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator and ensuring that the "wholes" are the same.

Common Misconceptions

» Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they may think that simply because .

» Students sometimes think, *"The longer the decimal number, the greater the value."* For example, they may think that 0.03 is greater than 0.3.

Domain: Measurement and Data

Measurement and Data
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. **M.4.19** Know relative sizes of measurement units within a system of units, including the metric system (km, m, cm; kg, g; l, ml), the standard system (lb., oz.), and time (hr., min, sec.). Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. (e.g., Know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …) **M.4.20** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Students will need ample opportunities to become familiar with new units of measure. In prior years, work with units was limited to units such as pounds, ounces, grams, kilograms, and liters, and students did not convert measurements.

Students may use two-column tables to convert from larger to smaller units and record equivalent measurements. For example:

kg	g
1	1000
2	2000
3	3000

ft	in
1	12
2	24
3	36

lb	oz
1	16
2	32
3	48

Students in grade four begin using the four operations to solve word problems involving measurement quantities such as liquid volume, mass, and time (**M.4.20**), including problems involving simple fractions or decimals.

Examples: Word Problems Involving Measures M.4.20

1. **Division/Fractions:** Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get? Students may record their solutions by using fractions of an inch or whole inches.
   Solution:
   The answer would be $\frac{2}{3}$ of a foot, or 8 inches. Students are able to express the answer in inches because they understand that $\frac{1}{3}$ of a foot is 4 inches and $\frac{2}{3}$ of a foot is 2 groups of $\frac{1}{3}$.

2. **Addition:** Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?
   Solution:
   Students know that 60 minutes make up one hour. We know Mason ran one hour, which is 60 minutes. He also ran 15 + 25 + 40 = 80 minutes more, which make 140 total minutes.

3. **Multiplication:** Mario and his two brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?
   Solution:
   Students know that 1 liter is 1000 milliliters (ml), so Mario brought 1000 + 500 = 1500 ml, and Javier brought 2 × 1000 = 2000 ml. This means the three brothers had a total of 1500 + 2000 + 450 = 3950 ml.

Adapted from ADE 2010.

Instructional Focus

In grade four, students use the four operations to solve word problems involving measurement quantities such as liquid volume, mass, and time (**M.4.19–M.4.20**). Measurement provides a context for solving problems using the four operations and connects to and supports major grade-level work in the cluster "Use the four operations with whole numbers to solve problems" (**M.4.1–M.4.3**) and clusters in the domain "Number and Operations—Fractions" (**M.4.12–M.4.15**). For example, students use whole-number multiplication to express measurements given in a larger unit in terms of a smaller unit, and they solve word problems involving addition and subtraction of fractions or multiplication of a fraction by a whole number.

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

M.4.21

Apply the area and perimeter formulas for rectangles in real-world and mathematical problems by viewing the area formula as a multiplication equation with an unknown factor. (e.g., find the width of a rectangular room given the area of the flooring and the length.)

In grade three, students develop an understanding of area and perimeter by using visual models. Students in grade four are expected to use formulas to calculate area and perimeter of rectangles; however, they still need to understand and be able to communicate their understanding of why the formulas work. It is still important for students to draw length units or square units inside a small rectangle make a visual distinction, and some students may still need to write the lengths of all four sides before finding the perimeter. Students know that answers for the area formula ($l \times w$) will be in square units and that answers for the perimeter formula ($2l + 2w$) or $2(l + w)$ will be in linear units (adapted from ADE 2010).

Example: Area and Perimeter of Rectangles M.4.21 (MHM2, MHM4)

Sally wants to build a pen for her dog, Callie. Her parents give her $200 to buy the fencing material, but they want Sally to design the pen. Her parents suggest that she consider different plans. Her parents also remind her that Callie needs as much room as possible to run and play, that the pen can be placed anywhere in the yard, and that the wall of the house could be used as one side of the pen. Sally decides to buy fencing material that costs $8.50 per foot. She also needs at least one three-foot-wide gate for the pen that costs $15.

- » Design a pen for Callie. Experiment with different pen designs and consider the advice from Sally's parents. Sally's house can be any configuration.
- » Write a letter to Sally with your diagrams and calculations. Explain why some designs are better for Callie than others.

Measurement and Data

Represent and interpret data.

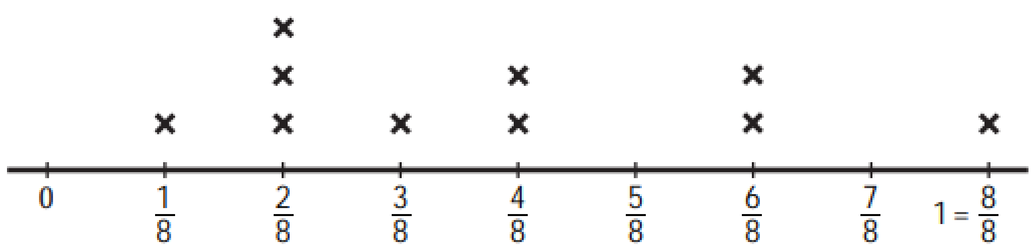
M.4.22

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots (e.g., from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection).

As students work with data in kindergarten through grade five, they build foundations for the study of statistics and probability in grades six and beyond, and they strengthen and apply what they learn in arithmetic.

Example: Interpreting Line Plots M.4.22

Ten students measure objects in their desk to the nearest inch. They record their results on the line plot below (in inches).



Possible related questions:

- » How many objects measured $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ inch?
- » If you put the objects end to end, what would the total length be?
- » If five $\frac{1}{8}$-inch pencils are placed end to end, what would the total length of the pencils be?

Adapted from ADE 2010.

Measurement and Data

Geometric measurement: understand concepts of angle and measure angles.

M.4.23

Recognize angles as geometric shapes that are formed when two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.

b. An angle that turns through b one-degree angles is said to have an angle measure of b degrees.

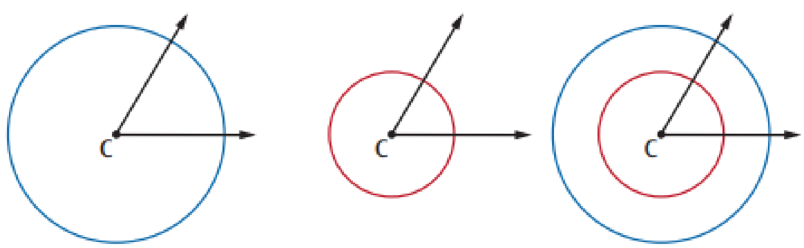
M.4.24

Measure angles in whole-number degrees using a protractor and sketch angles of specified measure.

M.4.25

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).

Students in grade four learn that angles are geometric shapes formed by two rays that share a common endpoint (**M.4.23**). They understand angle measure as being that portion of a circular arc that is formed by the angle where the shared vertex is the center of the circle. The figure below helps students see that an angle is determined by the arc it creates relative to the size of the entire circle. The diagrams below show angles having the same measure even though the circles are not the same size.

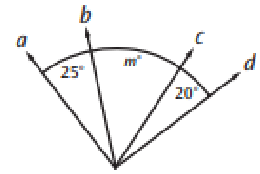


For example, the pie-shaped pieces formed by each angle are not the same size; this shows that angle measure is not defined in terms of these areas. The angle in each case measures . Since the total circumference of a circle is 360^0 and the angle measure is 60^0, the arc measures 60/360 or $\frac{1}{6}$ the total circumference of the circle in both the larger and smaller circles—but the pie-shaped pieces formed by the angle have different areas.

Before students begin measuring angles with protractors (**M.4.24**), they need to have some experience with benchmark angles. They transfer their understanding that a rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90° and 180°. They extend this understanding and recognize and sketch angles that measure approximately 45° and 30°. Students use appropriate terminology (*acute, right,* and *obtuse*) to describe angles and rays (*perpendicular*). Students recognize angle measure as additive and use this to solve addition and subtraction problems to find unknown angles on a diagram.

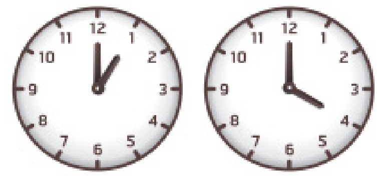
Examples: Angle measure is additive	M.4.25 (MHM1, MHM2, MHM4, MHM7)

1. If ray *a* is perpendicular to ray *d* (see **M.4.26**), what is the value of *m*?



   Solution:
   "Since perpendicular lines form an angle that measures 90°, I know that $25° + m° + 20° = 90°$. This means that $m° = 90° - 45° = 45°$."

2. Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30°. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?



   Solution:
   "This looks like it is four times as much, so it is $4 \times 30° = 120°$."

Adapted from ADE 2010.

Instructional Focus

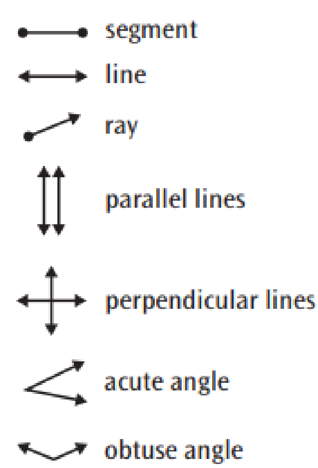
Students' work with concepts of angle measures (**M.4.23a, M.4.25**) also connects to and supports the addition of fractions, which is major work at the grade in the cluster "Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers" (**M.4.14–M.4.15**). For example, a 1° measure is a fraction of an entire rotation, and adding angle measures together is the same as adding fractions with a denominator of 360.

Before students solve word problems involving unknown angle measures (**M.4.25**), they need to understand concepts of angle measure (**M.4.23**) and gain some experience measuring angles (**M.4.24**). Students also need some familiarity with the geometric terms that are used to define angles as geometric shapes (**M.4.26**).

Domain: Geometry

A critical area of instruction in grade four is classifying and analyzing geometric figures based on their properties, such as parallel sides, perpendicular sides, particular angle measures, and symmetry.

Geometry
Draw and identify lines and angles and classify shapes by properties of their lines and angles.
M.4.26 Draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines. Identify these in two-dimensional figures.
M.4.27 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
M.4.28 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.



In grade four, students are exposed to the concepts of rays, angles, and perpendicular and parallel lines (**M.4.26**) for the first time. In addition, students classify figures based on the presence or on the absence of parallel or perpendicular lines and angles of a specified size (**M.4.27**). It is helpful to provide students with a visual reminder of examples of points, line segments, lines, angles, parallelism, and perpendicularity. For example, a wall chart with the images shown at right could be displayed in the classroom.

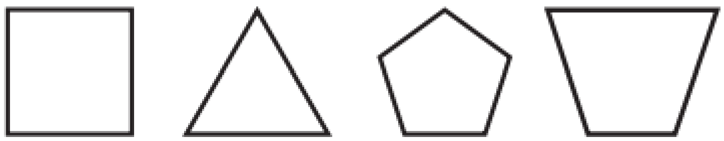
Students need to see all of these representations in different orientations. Students could draw these in different orientations and decide if all of the drawings are correct. They also need to see and draw a range of angles that are acute and obtuse.

Two-dimensional figures may be classified according to characteristics, such as the presence of parallel or perpendicular lines or by angle measurements. Students may use transparencies with lines drawn on them to arrange two lines in different ways to determine that the two lines might intersect at one point or might never intersect, thereby understanding the notion of parallel lines. Further investigations may be initiated with geometry software. These types of explorations may lead to a discussion on angles. (Patty paper is another option for comparing polygons.)

Students' prior experience with drawing and identifying right, acute, and obtuse angles helps them classify two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, 360° and to approximate the measurement of angles. Right triangles (triangles with one right angle) can be a category for classification, with subcategories—for example, an isosceles right triangle (a right triangle having two or more congruent sides) and a scalene right triangle (a right triangle with no congruent sides).

Examples: Classifying Shapes According to Attributes M.4.27 (MHM3)

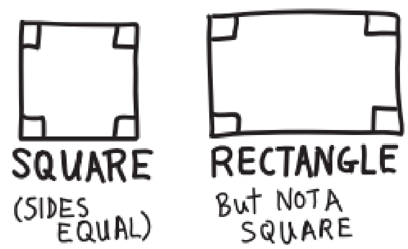
1. Identify which of the following shapes have perpendicular or parallel sides, and justify your selection(s).



Solution:
"I know that pairs of lines are perpendicular if they cross to form square corners or right angles. Lines are parallel if they are always the same distance apart and never cross each other. I compared pairs of line segments in each of the four figures and found the first figure includes both parallel and perpendicular lines, and the last figure includes one pair of parallel lines. The other two figures do not include either perpendicular or parallel lines."

2. Explain why a **square is considered a rectangle**, but a **rectangle is not necessarily a square**.

Solution:
"I know that rectangles are four-sided shapes that have four right angles. This makes any square a rectangle, since a square has four sides and four right angles also. But a square is a *special* kind of rectangle. What I mean is that you can have a rectangle that has its sides not all equal, and then it isn't a square. I drew examples to show what I mean."



Finally, students recognize a line of symmetry for a two-dimensional figure as a line across the figure, such that the figure can be folded along the line into matching parts (adapted from ADE 2010).

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals. There should be a balance of student experience working in the following areas: concepts, procedural skills, and problem solving. Arithmetic is viewed as an important set of skills and also as an analytical subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are connected specifically to arithmetic. Multiplication and division of whole numbers and fractions are instructional foci in grades three through five.

To be prepared for grade-five mathematics, students demonstrate that they have learned certain mathematical concepts and acquired procedural skills by the end of grade four and have met the fluency expectations for their grade level. For students in grade four, the expected fluencies are to add and subtract multi-digit whole numbers using the standard algorithm within 1,000,000 (**M.4.9**). These fluencies and the conceptual understandings that support them are foundational for work in later grades.

Of particular importance at grade four are concepts, skills, and understandings needed to use the four operations with whole numbers to solve problems (**M.4.1–M.4.3**); generalize place-value understanding for multi-digit whole numbers (**M.4.6–M.4.8**); use place-value understanding and properties of operations to perform multi-digit arithmetic (**M.4.9-M.4.11**); extend understanding of fraction equivalence and ordering (**M.4.12-M.4.13**); build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (**M.4.14–M.4.15**); and understand decimal notation for fractions and compare decimal fractions (**M.4.16–M.4.18**).

Fractions

Fraction equivalence is an important theme in the standards. Understanding fraction equivalence is necessary to extend arithmetic from whole numbers to fractions and decimals. Students need to understand fraction equivalence and that $\frac{a}{b} = \frac{n \times a}{n \times b}$. They should be able to represent equivalent common fractions and to apply this understanding when comparing fractions and when expressing fractional relationships using the symbols >, =, or <. Students understand how to represent and read fractions and mixed numbers in multiple ways.

Grade-four students should understand addition and subtraction with fractions having like denominators. This understanding represents a multi-grade progression, as students add and subtract fractions in grade four with like denominators by thinking of adding or subtracting so many unit fractions. Students should be able to solve word problems involving addition and subtraction of fractions that refer to the same whole and have like denominators (e.g., by using visual fraction models and equations to represent the problem). Students should understand how to add and subtract fractions and mixed numbers with like denominators.

Students further extend their understanding of multiplication to multiply fractions by whole numbers. To support their understanding, students should understand a fraction as the numerator times the unit fraction with the same denominator. Students should be able to rewrite fractions as multiples of the unit fraction of the same denominator, use a visual model to multiply a fraction by a whole number, and use equations to represent problems involving the multiplication of a fraction by a whole number by multiplying the whole number times the numerator of the fraction.

Four Operations with Whole Numbers

By the end of grade four, students should fluently add and subtract multi-digit whole numbers to 1,000,000 using the standard algorithm. Students should also be able to use the four operations to solve multi-step word problems with whole-number remainders.

In grade four, students develop their understanding and skills with multiplication and division. They combine their understanding of the meanings and properties of multiplication and division with their understanding of base-ten units to begin to multiply and divide multi-digit numbers. Students in grade four should know how to express the product of two multi-digit numbers as another multi-digit number. They also should know how to find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Using a rectangular area model to represent multiplication and division helps students visualize these operations. This work will develop further in grade five and culminates in fluency with the standard algorithms in grade six.

Illustrative Mathematics
High-quality educational resources for Mathematics, supporting the teacher's content knowledge.
https://www.illustrativemathematics.org/

Making Math Centers Work
Math centers, like any other strategy, has to be personalized by the teacher to best fit the students' needs and teacher's style. The following videos walk the viewer through a variety of strategies for math centers.
http://okmathteachers.com/making-math-centers-work-for-you/

Quantile Teacher Assistant
A tool aligned to WV College- and Career-Readiness Standards for Mathematics designed to help educators locate resources that can support instruction and identify skills most relevant to standards.
https://math-tools.quantiles.com/quantile-teacher-assistant/

WV Teacher Resources for Educational Excellence (WV TREE)
Curated resources at specific grade levels for WV early and elementary teachers.
https://wvde.us/tree/

Supplemental Instructional Materials for Mathematics

Number Talks Fractions, Decimals, and Percentages by Sherry Parrish, Ann Dominick, Steve Leinwand

Professional Learning

Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching by Jo Boaler, Carol Dweck

W. Clayton Burch
West Virginia Superintendent of Schools