West Virginia math4life

Educators' Guide for Mathematics

Grade 2

West Virginia DEPARTMENT OF EDUCATION

Grade Two

Mathematics Instruction

In prior grades, students developed a foundation for understanding place value, including grouping in tens and ones. In grade one, they built understanding of whole numbers to 120 and developed strategies to add, subtract, and compare numbers. They solved addition and subtraction word problems within 20 and developed fluency with these operations within 10. Students also worked with non-standard measurement and reason about attributes of geometric shapes (adapted from Charles A. Dana Center 2012).

In grade two, instructional time focuses on four critical areas: (1) extending understanding of base ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO] 2010i). Students also work toward fluency with addition and subtraction within 20 using mental strategies and within 100 using strategies based on place value, properties of operations, and the relationship between addition and subtraction. They know from memory all sums of two one-digit numbers.

West Virginia College- and Career-Readiness Standards for Mathematics

The West Virginia College- and Career-Readiness Standards for Mathematics (WVBE Policy 2520.2B) emphasize key content, skills, and practices at each grade level and support three major principles:

- Instruction is focused on grade level standards.
- Instruction is attentive to learning across grades and to linking major topics within grades.
- Instruction develops conceptual understanding, procedural skill and fluency, and application.

Grade level examples of these three major principles are indicated throughout this document.

Cluster headings can be viewed as the most effective way to communicate the focus and coherence of the standards. The instructional focus must be based on the depth of the ideas, the time needed to master the clusters, and their importance to future mathematics or the later demands of preparing for college and careers.

Teachers and administrators alike understand that the standards are not topics to be checked off after being covered in isolated units of instruction; rather, they provide content to be developed throughout the school year through rich instructional experiences presented in a coherent manner. West Virginia College- and Career-Readiness Standards for Mathematics are learning goals for students that must be mastered by the end of the grade two academic year in order for students to be prepared for the third grade mathematics content.

Mathematical Fluency

Students demonstrate fluency of mathematical standards when they exhibit the following:

- Accuracy – ability to produce an accurate answer
- Efficiency – ability to choose an appropriate expedient strategy for a specific computation problem
- Flexibility – ability to use number relationships with ease in computation.

Connecting Mathematical Habits of Mind and Content

The Mathematical Habits of Mind (MHM) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics in the classroom. The MHM are the behaviors and dispositions of mathematics and should be integrated into every mathematics lesson for all students, and are part of the comprehensive approach to early and elementary learning per *WVBE Policy, 2510, Assuring Quality of Education: Regulations for Education Programs*.

Although the description of the Mathematical Habits of Mind remains the same at all grade levels, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. The following chart presents examples of how the Mathematical Habits of Mind may be integrated into tasks appropriate for students in grade two.

Mathematical Habits of Mind—Explanation and Examples for Grade Two

Mathematical Habits of Mind	Explanation and Examples
MHM1 Make sense of problems and persevere in solving them.	In grade two, students realize that doing mathematics involves reasoning about and solving problems. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They make conjectures about the solution and plan out a problem-solving approach.
MHM2 Reason abstractly and quantitatively.	Students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Students represent situations by decontextualizing tasks into numbers and symbols. For example, a task may be presented as follows: “There are 25 children in the cafeteria, and they are joined by 17 more children. How many students are in the cafeteria?” Students translate the situation into an equation (such as 25 + 17 = __) and then solve the problem. Students also contextualize situations during the problem-solving process. To reinforce students’ reasoning and understanding, teachers might ask, “How do you know?” or “What is the relationship of the quantities?”

Mathematical Habits of Mind	Explanation and Examples
MHM3 Construct viable arguments and critique the reasoning of others.	Grade two students may construct arguments using concrete referents, such as objects, pictures, math drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions such as “How did you get that?”, “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but also listen to others’ explanations. They decide if the explanations make sense and ask appropriate questions. Students critique the strategies and reasoning of their classmates. For example, to solve 74 – 18, students might use a variety of strategies and discuss and critique each other’s reasoning and strategies.
MHM4 Model with mathematics.	Students experiment with representing problem situations in multiple ways, including writing numbers, using words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, or creating equations. Students need opportunities to connect the different representations and explain the connections. Students model real-life mathematical situations with an equation and check to make sure that their equation accurately matches the problem context. They use concrete manipulatives or math drawings (or both) to explain the equation. They create an appropriate problem situation from an equation. For example, students create a story problem for the equation 43 + ____ = 82, such as “There were 43 mini-balls in the machine. Tom poured in some more mini-balls. There are 82 mini-balls in the machine now. How many balls did Tom pour in?” Students are encouraged to answer questions, such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?”
MHM5 Use appropriate tools strategically.	In grade two, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited than others. For instance, grade two students may decide to solve a problem by making a math drawing rather than writing an equation. Students may use tools such as snap cubes, place value (base ten) blocks, hundreds number boards, number lines, rulers, virtual manipulatives, diagrams, and concrete geometric shapes (e.g., pattern blocks, three-dimensional solids). Students understand which tools are the most appropriate to use. For example, while measuring the length of the hallway, students are able to explain why a yardstick is more appropriate to use than a ruler. Students should be encouraged to answer questions such as, “Why was it helpful to use ________?”

Mathematical Habits of Mind	Explanation and Examples
MHM6 Attend to precision.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning. Students communicate clearly, using grade level appropriate vocabulary accurately and precise explanations and reasoning to explain their process and solutions. For example, when measuring an object, students carefully line up the tool correctly to get an accurate measurement. During tasks involving number sense, students consider if their answers are reasonable and check their work to ensure the accuracy of solutions.
MHM7 Look for and make use of structure.	Grade two students look for patterns and structures in the number system. For example, students notice number patterns within the tens place as they connect counting by tens to corresponding numbers on a hundreds chart. Students see structure in the base ten number system as they understand that 10 ones equal a ten, and 10 tens equal a hundred. Teachers might ask, "What do you notice when ________?" or "How do you know if something is a pattern?" Students adopt mental math strategies based on patterns (making ten, fact families, doubles). They use structure to understand subtraction as an unknown addend problem (e.g., 50 – 33 = __ can be written as 33 + __ = 50 and can be thought of as "How much more do I need to add to 33 to get to 50?").
MHM8 Look for and express regularity in repeated reasoning.	Second-grade students notice repetitive actions in counting and computation (e.g., number patterns to count by tens or hundreds). Students continually check for the reasonableness of their solutions during and after completion of a task by asking themselves, "Does this make sense?" Students are encouraged to answer questions — such as "What is happening in this situation?" or "What predictions or generalizations can this pattern support?"

Adapted from Arizona Department of Education (ADE) 2010 and North Carolina Department of Public Instruction (NCDPI) 2013b.

Standards-Based Learning at Grade Two

The following narrative is organized by the domains in the West Virginia College- and Career-Readiness Standards for Mathematics and highlights some necessary foundational skills from previous grade levels. It also provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind (**MHM**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application.

Domain: Operations and Algebraic Thinking

A critical area of instruction in grade two is building fluency with addition and subtraction. Grade two students fluently add and subtract within 20 and solve addition and subtraction word problems involving unknown quantities in all positions within 100. Grade two students also work with equal groups of objects to gain the foundations for multiplication.

Operations and Algebraic Thinking
Represent and solve problems involving addition and subtraction. **M.2.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g. by using drawings and equations with a symbol for the unknown number to represent the problem).

In grade two, students add and subtract numbers within 100 in the context of one- and two-step word problems (**M.2.1**). By grade two, students have worked with various problem situations (add to, take from, put together, take apart, and compare) with unknowns in all positions (result unknown, change unknown, and start unknown). Grade two students extend their work with addition and subtraction word problems in two significant ways:

- They represent and solve problems of all types involving addition and subtraction within 100, building upon their previous work within 20.
- They represent and solve two-step word problems of all types, extending their work with one-step word problems (adapted from ADE 2010; NCDPI 2013b; Georgia Department of Education [GaDOE] 2011; and Kansas Association of Teachers of Mathematics [KATM] 2012, 2nd Grade Flipbook).

Different types of addition and subtraction problems are presented in the following chart.

Types of Addition and Subtraction Problems (Grade Two)

	Result Unknown	Change Unknown	Start Unknown
Add to	There are 22 marbles in a bag. Thomas placed 23 more marbles in the bag. How many marbles are in the bag now? 22 + 23 = ☐	Bill had 25 baseball cards. His mom gave him some more. Now he has 73 baseball cards. How many baseball cards did his mom give him? In this problem, the starting quantity is provided (25 baseball cards), a second quantity is added to that amount (some baseball cards), and the result quantity is given (73 baseball cards). This question type is more algebraic and challenging than a “result unknown” problem and can be modeled by a situational equation (25 +☐= 73) that does not immediately lead to the answer. Students can write a related equation (73 – 25 = ☐)—called a solution equation—to solve the problem.	Some children were playing on the playground, and 5 more children joined them. Then there were 22 children. How many children were playing before? This problem can be represented by ☐+ 5 = 22. The “start unknown” problems are difficult for students to model because the initial quantity is unknown, and therefore some students do not know how to start a solution strategy. They can make a drawing, where it is crucial that they realize that the 5 is part of the 22 total children. This leads to more general solutions by subtracting the known addend or counting/adding on from the known addend to the total.
Take from	There were 45 apples on the table. I took 12 of those apples and placed them in the refrigerator. How many apples are on the table now? 45 – 12 = ☐	Andrea had 51 stickers. She gave away some stickers. Now she has 22 stickers. How many stickers did she give away? This question may be modeled by a situational equation (51 – ☐= 22) or a solution equation (51 – 22 = ☐). Both the “take from” and “add to” questions involve actions.	Some children were lining up for lunch. After 4 children left, there were 26 children still waiting in line. How many children were there before? This problem can be modeled by ☐ – 4 = 26. Similar to the previous “add to (with start unknown)” problem, “take from (with start unknown)” problems require a high level of conceptual understanding.

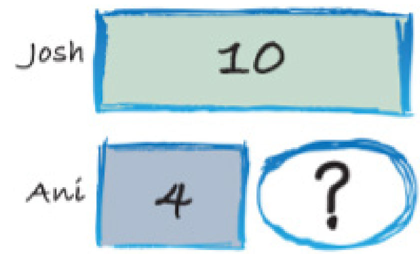
	Total Unknown	Addend Unknown	Both Addends Unknown
Put together/ Take apart	There are 30 red apples and 20 green apples on the table. How many apples are on the table? 30 + 20 = ?	Roger puts 24 apples in a fruit basket. Nine (9) are red and the rest are green. How many are green?” There is no direct or implied action. The problem involves a set and its subsets. It may be modeled by 24 – 9 = □ or 9 + □= 24. This type of problem provides students with opportunities to understand subtraction as an unknown-addend problem.	Grandma has 5 flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	Pat has 19 peaches. Lynda has 14 peaches. How many more peaches does Pat have than Lynda? “Compare” problems involve relationships between quantities. Although most adults might use subtraction to solve this type of problem (19 – 14 = □), students will often solve this problem as an unknown-addend problem (14 + □= 19) by using a counting-up or matching strategy. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context—not the representation separated from its context.	**(“More” version):** Theo has 23 action figures. Rosa has 2 more action figures than Theo. How many action figures does Rosa have? This problem can be modeled by 23 + 2 = □. **(“Fewer” version):** Lucy has 28 apples. She has 2 fewer apples than Marcus. How many apples does Marcus have? This problem can be modeled as 28 + 2 = □. The misleading language form “fewer” may lead students to choose the wrong operation. Care must be taken to avoid promoting a reliance on key words rather than an understanding of the context of the problem.	**(“More” version):** David has 27 more bunnies than Keisha. David has 28 bunnies. How many bunnies does Keisha have? This problem can be modeled by 28 – 27 = □. The misleading language form “more” may lead students to choose the wrong operation. Care must be taken to avoid promoting a reliance on key words rather than an understanding of the context of the problem. **(“Fewer” version):** Bill has 24 stamps. Lisa has 2 fewer stamps than Bill. How many stamps does Lisa have? This problem can be modeled as 24 – 2 = □.

For these more complex grade two problems, it is crucial for students to represent the problem situations with drawings and equations (**M.2.1**). Drawings can be shown more easily to the whole class during explanations and can be related to equations. Students can also use manipulatives (e.g., snap cubes, place value blocks), but drawing quantities is an exercise that can be used anywhere to solve problems and support students in describing their strategies. Grade two students represent problems with equations and use boxes, blanks, or pictures for the unknown amount. For example, students can represent “compare” problems using comparison bars (see figure 2-1). Students can draw these bars, fill in numbers from the problem, and label the bars. Using comparison bars supports the introduction of tape diagrams.

Figure 2-1. Comparison Bars

Josh has 10 markers, and Ani has 4 markers. How many more markers does Josh have than Ani?



Example **M.2.1**

10	
4	?

One-step word problems use one operation. Two-step word problems (**M.2.1**) are new for grade two and require students to complete two operations, which may include the same operation or different operations.

Initially, two-step problems do not involve the most difficult subtypes of problems (e.g., “compare” and “start unknown” problems) and are limited to single-digit addends. There are many problem-situation subtypes and various ways to combine such subtypes to devise two-step problems. Introducing easier problems first will provide support for grade two students who are still developing proficiency with “compare” and “start unknown” problems.

The following table presents examples of easy and moderately difficult two-step word problems that would be appropriate for grade two students.

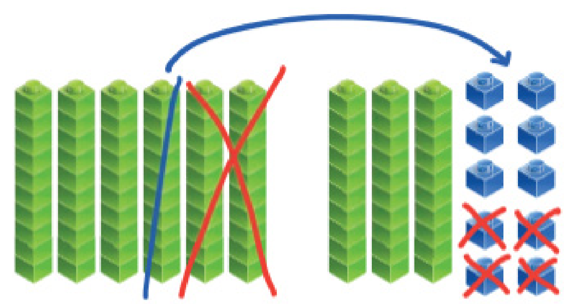
One-Step Word Problem One Operation	Two-Step Word Problem Two Operations, Same	Two-Step Word Problem Two Operations, Opposite
There are 15 stickers on the page. Brittany put some more stickers on the page and now there are 22. How many stickers did Brittany put on the page? 15+☐=22 or 22-15=☐	There are 9 blue marbles and 6 red marbles in the bag. Maria put in 8 more marbles. How many marbles are in the bag now? 9+6+8=☐ or (9+6)+8=☐	There are 39 peas on the plate. Carlos ate 25 peas. Mother put 7 more peas on the plate. How many peas are on the plate now? 39-25+7=☐ or (39-25)+7=☐

Grade two students use a range of methods, often mastering more complex strategies such as making tens and doubles and near doubles that were introduced in grade one for problems involving single-digit addition and subtraction. Grade two students also begin to apply their understanding of place value to solve problems, as shown in the following example.

One-Step Problem: Some students are in the cafeteria. Twenty-four (24) more students came in. Now there are 60 students in the cafeteria. How many students were in the cafeteria to start with? Use drawings and equations to show your thinking.

Student A:
I read the problem and thought about how to write it with numbers. I thought, "What and 24 makes 60?" I used a math drawing to solve it. I started with 24. Then I added tens until I got close to 60; I added 3 tens. I stopped at 54. Then I added 6 more ones to get to 60. So, 10 + 10 + 10 + 6 = 36. So, there were 36 students in the cafeteria to start with. My equation for the problem is ☐ + 24 = 60 . (**MHM2, MHM7, MHM8**)

Student B:
I read the problem and thought about how to write it with numbers. I thought, "There are 60 total. I know about the 24. So, what is 60 – 24?" I used place value blocks to solve it. I started with 60 and took 2 tens away. I needed to take 4 more away. So, I broke up a ten into 10 ones. Then I took 4 away. That left me with 36. So, 36 students were in the cafeteria at the beginning.
60 – 24 = 36.
My equation for the problem is 60 – 24 = ☐.
(MHM2, MHM4, MHM5, MPHM6)



Adapted from ADE 2010, NCDPI 2013b, GaDOE 2011, and KATM 2012 (2nd Grade Flipbook)

As students solve addition and subtraction word problems, they use concrete manipulatives, pictorial representations, and mental mathematics to make sense of a problem (**MHM1**); they reason abstractly and quantitatively as they translate word problem situations into equations (**MHM2**); and they model with mathematics (**MHM4**).

The following chart presents a sample classroom activity that connects the Standards for Mathematical Content and the Mathematical Habits of Mind.

To solve word problems, students learn to apply various computational methods. Kindergarten students generally use Level 1 methods, and students in grades one and two use Level 2 and Level 3 methods. The three levels are summarized in the chart below.

Methods Used for Solving Single-Digit Addition and Subtraction Problems
Level 1: Direct Modeling by Counting All or Taking Away Represent the situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend. **Level 2: Counting On** Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total, but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. The count is tracked and monitored in some way (e.g., with fingers, objects, mental images of objects, body motions, or other count words). For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as the unknown addend). **Level 3: Converting to an Easier Equivalent Problem** Decompose an addend and compose a part with another addend.

Adapted from UA Progressions Documents 2011a

Grade two students who continue to rely on the Level 1 strategy of Counting All to solve problems need to be provided additional support so they are able to develop Level 2 and Level 3 strategies to solve problems.

In grade two, students extend their fluency with addition and subtraction from within 10 to within 20 (**M.2.2**). The experiences students have had with addition and subtraction in kindergarten (within 5) and grade one (within 10) culminate in grade two students becoming fluent in single-digit additions and related subtractions, using Level 2 and Level 3 methods and strategies as needed.

Operations and Algebraic Thinking
Add and subtract within 20. **M.2.1** Fluently add and subtract within 20 using mental strategies and by end of Grade 2, know from memory all sums of two one-digit numbers.

See standard M.1.6 for a list of mental strategies.

Students may still need to support the development of their fluency with math drawings when solving problems. Math drawings represent the number of objects counted (using dots and sticks) and do not need to represent the context of the problem. Thinking about numbers by using 10-frames or making drawings using groups of fives and tens may be helpful ways to understand single-digit additions and subtractions. The National Council of Teachers of Mathematics Illuminations project (NCTM Illuminations 2013a) offers examples of interactive games that students can play to develop counting and addition skills.

FLUENCY
(K–6) set expectations for fluency in computation (e.g., "Fluently add and subtract within 20 . . .") [M.2.2]. Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. The word fluent is used in the standards to mean "reasonably fast and accurate" and possessing the ability to use certain facts and procedures with enough facility that using such knowledge does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of knowing some answers, knowing some answers from patterns, and knowing some answers through the use of strategies. *Adapted from UA Progressions Documents 2011a.*

Mental strategies, such as those listed in the following table, help students develop fluency in adding and subtracting within 20 as they make sense of number relationships. The following table presents the mental strategies listed with standard **M.1.6** as well as two additional strategies.

Mental Strategies

- Counting on
- Making tens (9 + 7 = [9 + 1] + 6 = 10 + 6)
- Decomposing a number leading to a ten (14 – 6 = 14 – 4 – 2 = 10 – 2 = 8)
- Related facts (8 + 5 = 13 and 13 – 8 = 5)
- Doubles (1 + 1, 2 + 2, 3 + 3, and so on)
- Doubles plus one (7 + 8 = 7 + 7 + 1)
- Relationship between addition and subtraction (e.g., by knowing that 8 + 4 = 12, one also knows that 12 – 8 = 4)
- Equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13)

Adapted from NCDPI 2013b.

Grade two students build important foundations for multiplication as they explore odd and even numbers in a variety of ways (**M.2.3**). They use concrete objects (e.g., counters or place value cubes) and move toward pictorial representations such as circles or arrays (**MHM1**). Through investigations, students realize that an even number of objects can be separated into two equal groups (without extra objects remaining), while an odd number of objects will have one object remaining (**MHM7** and **MHM8**).

Operations and Algebraic Thinking

Work with equal groups of objects to gain foundations for multiplication.

M.2.3

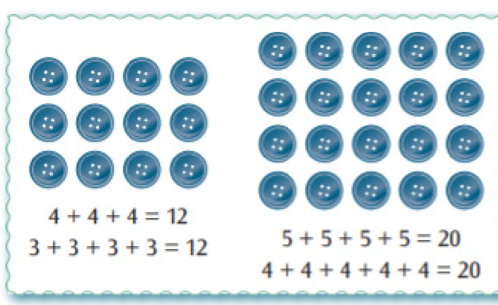
Determine whether a group of objects (up to 20) has an odd or even number of members, e.g. by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

M.2.4

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Students also apply their work with doubles addition facts and decomposition of numbers (breaking them apart) into two equal addends (e.g., 10 = 5 + 5) to understand the concept of even numbers. Students reinforce this concept as they write equations representing sums of two equal addends, such as 2 + 2 = 4, 3 + 3 = 6, 5 + 5 = 10, 6 + 6 = 12, or 8 + 8 = 16. Students are encouraged to explain how they determined if a number is odd or even and what strategies they used (**MHM3**).

With standard **M.2.4**, grade two students use rectangular arrays to work with repeated addition—an important building block for multiplication in grade three—using concrete objects (e.g., counters, buttons, square tiles) as well as pictorial representations on grid paper or other drawings of arrays (**MHM1**). To lay the foundation for multiplicative understanding, the use of arrays in addition provides a visual reference for students to use



that will promote the connection between addition and multiplication in grade three. Using the commutative property of multiplication, students add either the rows or the columns and arrive at the same solution (**MHM2**). Students write equations that represent the total as the sum of equal addends, as shown in the examples at the right.

The first example helps lead students to later understanding that 3×4=4×3; the second example supports the fact that 4×5=5×4 (ADE 2010). This understanding is built through the ideas presented in the previous paragraph.

Instructional Focus
In the cluster "Work with equal groups of objects to gain foundations for multiplication," student work reinforces addition skills and understandings and is connected to work in the clusters "Represent and solve problems involving addition and subtraction" (**M.2.1**) and "Add and subtract within 20" (**M.2.2**). Also, as students work with odd and even groups (**M.2.3**) they build a conceptual understanding of equal groups, which supports their introduction to multiplication and division in grade three.

Domain: Number and Operations in Base Ten

A critical area of instruction in grade two is to extend students' understanding of base ten notation to include hundreds. Grade two students understand multi-digit numbers (up to 1000). They add and subtract within 1000 and become fluent with addition and subtraction within 100 using place value strategies (UA Progressions Documents 2012b).

Number and Operations in Base Ten

Understand place value.

M.2.5

Understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones (e.g., 706 equals 7 hundreds, 0 tens and 6 ones). Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens – called a "hundred."

b. Numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight or nine hundreds, and 0 tens and 0 ones.

M.2.6

Count within 1000 and skip-count by 5s, 10s and 100s.

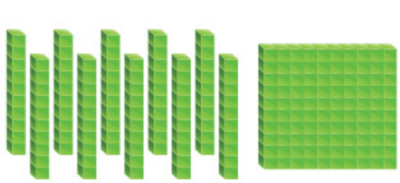
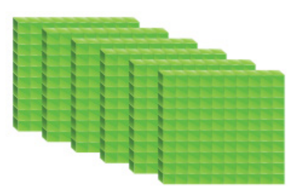
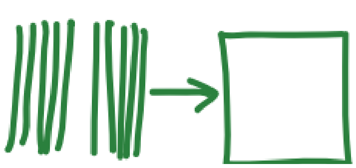
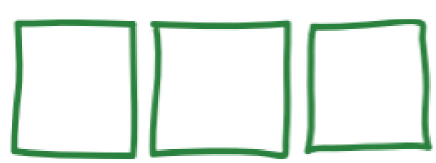
M.2.7

Read and write numbers to 1000 using base-ten numerals, number names and expanded form.

M.2.8

Compare two three-digit numbers based on meanings of the hundreds, tens and ones digits, using >, = and < symbols to record the results of comparisons.

Grade two students build on their previous work with groups of tens to make bundles of hundreds, with or without leftovers, using base ten blocks, cubes in towers of 10, 10-frames, and so forth, as well as math drawings that initially show the 10 tens within 1 hundred, but then move to a quick-hundred version that is a drawn square in which students visualize 10 tens; see the table below for examples. Bundling hundreds will support students' discovery of place value patterns (**MHM7**) Students explore the idea that numbers such as 100, 200, 300, and so on are groups of hundreds that have "0" in the tens and ones places. Students might represent numbers using place value (base ten) blocks or math drawings (**MHM1**).

Using Base Ten Blocks	**M.2.5**
These have the same value: 	Six (6) hundreds is the same as 600: 
Using Math Drawings	
When I bundle 10 "ten-sticks," I get 1 "hundred square." 	The picture shows 3 hundreds, or 300. 

As students represent various numbers, they associate number names with number quantities (**MHM2**). For example, 243 can be expressed as both "2 groups of hundred, 4 groups of ten, and 3 ones" and "24 tens and 3 ones." Students can read number names as well as place value concepts to say a number. For example, 243 may be read as "two hundred forty-three" as well as "2 hundreds, 4 tens, and 3 ones." Flexibility with seeing a number like 240 as "2 hundreds and 4 tens" as well as "24 tens" is an important indicator of place value understanding (KATM 2012, 2nd Grade Flipbook).

In kindergarten, students were introduced to counting by tens. In grade two they extend this to skip-count by twos, fives, tens, and hundreds (**M.2.6**). Exploring number patterns can help students skip-count. For example, when skip-counting by fives, the ones digit alternates between 5 and 0, and when skip-counting by tens and hundreds, only the tens and hundreds digits change, increasing by one each time. In this way, skip-counting can reinforce students' understanding of place value. Work with skip-counting lays a foundation for multiplication; however, because students do not keep track of the number of groups they have counted, they are not yet learning true multiplication. The ultimate goal is for grade two students to count in multiple ways without visual support.

Instructional Focus
As students explore number patterns by skip-counting, they also develop mathematical practices such as understanding the meaning of written quantities (**MHM2**) and recognizing number patterns and structures in the number system (**MHM7**).

Grade two students need opportunities to read and represent numerals in various ways (**M.2.7**). An example adapted from KATM (2012, 2nd Grade Flipbook) illustrates different ways for grade two students to represent numerals:

- Standard form (e.g., 637)
- Base ten numerals in standard form (e.g., 6 hundreds, 3 tens, and 7 ones)
- Number names in word form (e.g., six hundred thirty-seven)
- Expanded form (e.g., 600 + 30 + 7)
- Equivalent representations (e.g., 500 + 130 + 7; 600 + 20 + 17; 30 + 600 + 7)

When students read the expanded form for a number, they might say "6 hundreds plus 3 tens plus 7 ones" or "600 plus 30 plus 7." Understanding the expanded form is valuable when students use place value strategies to add and subtract large numbers (see also M.2.11).

Grade two students use the symbols for greater than (>), less than (<), and equal to (=) to compare numbers within 1000 (**M.2.8**). Students build on work in standards (**M.2.5** and **M.2.7**) by examining the amounts of hundreds, tens, and ones in each number. To compare numbers, students apply their understanding of place value. The goal is for students to understand that they look at the numerals in the hundreds place first, then the tens place, and if necessary, the ones place. It is important that students have multiple experiences communicating their comparisons in words before using only symbols to indicate greater than, less than, and equal to.

Example: Compare 452 and 455.	**M.2.8**
Student 1 explains that 452 has 4 hundreds, 5 tens, and 2 ones and that 455 has 4 hundreds, 5 tens, and 5 ones: "They have the same number of hundreds and the same number of tens, but 455 has 5 ones and 452 only has 2 ones. So, 452 is less than 455, or 452 < 455."	
Student 2 might think that 452 is less than 455: "I know this because when I count up, I say 452 before I say 455."	

Adapted from KATM 2012 (2nd Grade Flipbook).

As students compare numbers, they also develop mathematical practices such as making sense of quantities (**MHM2**), understanding the meaning of symbols (**MHM6**), and making use of number patterns and structures in the number system (**MHM7**).

Number and Operations in Base Ten
Use place value understanding and properties of operations to add and subtract.
M.2.9 Fluently add and subtract within 100 using strategies based on place value, properties of operations and/or the relationship between addition and subtraction.
M.2.10 Add up to four two-digit numbers using strategies based on place value and properties of operations.
M.2.11 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones and sometimes it is necessary to compose or decompose tens or hundreds.
M.2.12 Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900.
M.2.13 Explain why addition and subtraction strategies work, using place value and the properties of operations. Instructional Note: Explanations may be supported by drawing or objects.

Place value understanding is central to multi-digit computations. In grade two, students develop, discuss, and later use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base ten notation. While students become fluent in such methods within 100 at grade two, they also use these methods for sums and differences within 1000 (**M.2.9–13**).

Written methods for recording addition and subtraction are based on two important features of the base ten number system:

- When numbers are added or subtracted in the base ten system, like units are added or subtracted (e.g., ones are added to ones, tens to tens, hundreds to hundreds).
- Adding and subtracting multi-digit numbers written in base ten can be facilitated by composing and decomposing units appropriately, so as to reduce the calculations to adding and subtracting within 20 (e.g., 10 ones make 1 ten, 100 ones make 1 hundred, 1 hundred makes 10 tens).

As these strategies are merely extensions of those for numbers within 100, they can and should be applied for adding and subtracting numbers within 1000. Of course, all methods for adding and subtracting two- and three-digit numbers are based on place value and should be learned by students with an emphasis on understanding. Math drawings can support student understanding, and as students become familiar with math drawings, these drawings should accompany written methods.

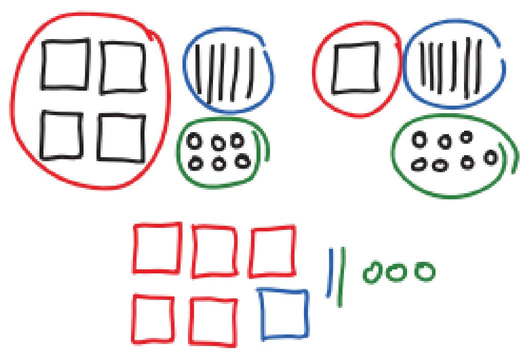
Addition

Figure 2-3 presents two written methods for addition, with accompanying illustrations (base ten blocks can also be used to illustrate). Students initially work with math drawings or manipulatives alongside the written methods, but they will eventually use written methods exclusively, mentally constructing pictures as necessary and using other strategies. Teachers should note the importance of these written methods as students generalize to larger numbers and decimals and emphasize the regrouping nature of combining units. These two methods are given only as examples and are not meant to represent all such place value methods.

Figure 2-3. Addition Methods Supported with Math Drawings

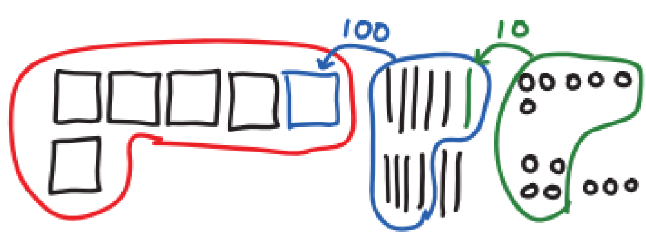
Examples	**M.2.11**

Addition Method 1: In this written addition method, all partial sums are recorded underneath the addition bar. Addition is performed from left to right in this example, but students can also work from right to left. In the accompanying drawing, it is clear that hundreds are added to hundreds, tens to tens, and ones to ones, which are eventually grouped into larger units where possible to represent the total, 623.



456 + 167 ———	456 + 167 ——— 500	456 + 167 ——— 500 110	456 + 167 ——— 500 110 13 ——— 623

Addition Method 2: In this written addition method, digits representing newly composed units are placed below the addends from which they were derived, to the right to indicate that they are represented as a larger, newly composed unit. The addition proceeds right to left. The advantage to placing the composed units as shown is that it is clearer where they came from—e.g., the 1 and 3 that came from the sum of the ones-place digits (6 + 7) are close to each other. This eliminates confusion that can arise from traditional methods involving “carrying,” which tends to separate the two digits that came from 13 and obscure the meaning of the numbers.



456 + 167 ———	456 + 167 1 ——— 3	456 + 167 1 1 ——— 23	456 + 167 1 1 ——— 623
	Add the ones, 6 + 7, and record these as 13, with 3 in the ones place and a 1 underneath the tens column	Add the tens, 5 + 6 + 1, and record these 12 tens with 2 in the tens place and 1 under the hundreds column.	Add the hundreds, 4 + 1 + 1, and record these 6 hundreds in the hundreds column.

Subtraction

In grade one, students were not expected to compute differences of two-digit numbers other than multiples of 10. In grade two, students subtract two-digit numbers, with and without decomposing, which highlights the similarity between these two cases.

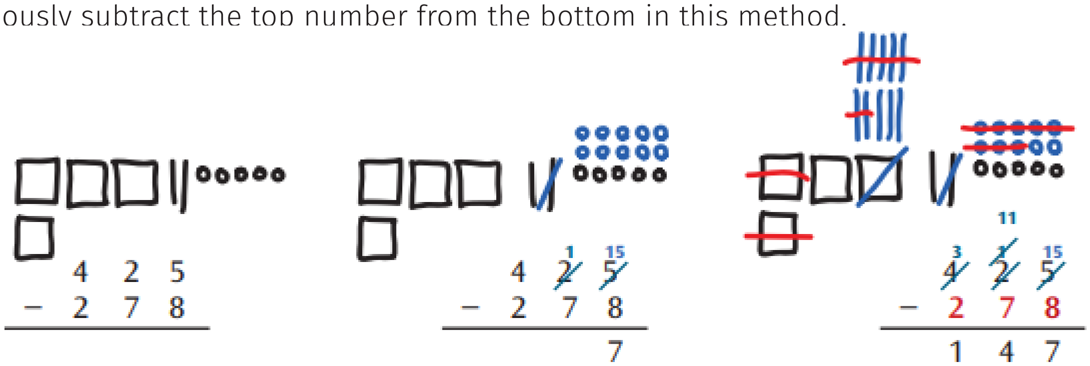
Figure 2-4 presents two methods for subtraction, one where all decomposing is done first, the other where decomposing is done as needed. Students will encounter situations in which they “don’t have enough” to subtract. This is more precise than saying, “You can’t subtract a larger number from a smaller number,” or the like, as the latter assertion is a false mathematical statement. In later grades, students will subtract larger numbers from smaller ones, and that will result in negative numbers as answers (for example, 9 – 15 = -6).

Note that the accompanying illustrations show the decomposing steps in each written subtraction method. Again, these methods generalize to numbers of all sizes and are based on decomposing larger units into smaller units when necessary.

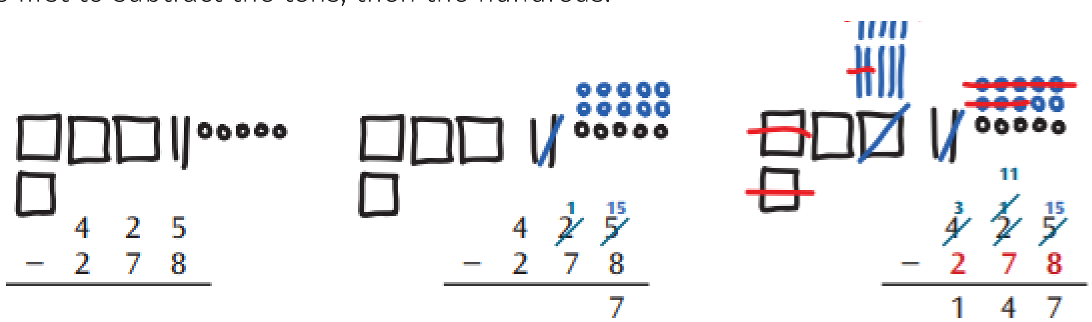
Figure 2-4. Subtraction Methods Supported with Math Drawings

Examples **M.2.11**

Subtraction Method 1: In this written subtraction method, all necessary decompositions are done first. Decomposing can start from the left or the right with this method. Students may be less likely to erroneously subtract the top number from the bottom in this method.



Subtraction Method 2: In this written subtraction method, decomposing is done as needed. Students first ungroup a ten so they can subtract 8 from 15. They may erroneously try to subtract the tens as well, getting 7 – 1 = 6. Led to see their error, students find they need to ungroup hundreds first to subtract the tens, then the hundreds.



Adapted from Fuson and Beckmann 2013 and UA Progressions Documents 2012b.

When developing fluency with adding and subtracting within 100 (**M.2.9**), grade two students use the methods just discussed, as well as other strategies, without the support of drawings.

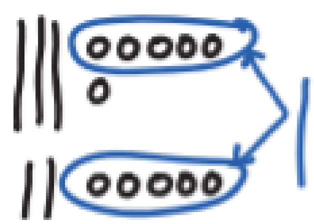
Strategies for Addition and Subtraction
Examples of addition strategies based on place value for 48 + 37: » Adding by place value: 40 + 30 = 70, 8 + 7 = 15, and 70 + 15 = 85 » Incremental adding (by tens and ones): 48 + 10 = 58, 58 + 10 = 68, 68 + 10 = 78, and 78 + 7 = 85 » Composing and decomposing (making a "friendly" number): 48 + 2 = 50, 37 – 2 = 35, and 50 + 35 = 85 **Examples of subtraction strategies based on place value for 81 – 37:** » Adding up (from smaller number to larger number): 37 + 3 = 40, 40 + 40 = 80, 80 + 1 = 81, and 3 + 40 + 1 = 44 » Incremental subtracting: 81 – 10 = 71, 71 – 10 = 61, 61 – 10 = 51, 51 – 7 = 44 » Subtracting by place value: 81 – 30 = 51, 51 – 7 = 44 *Adapted from ADE 2010.*

As students develop fluency with adding and subtracting within 100, they also support mathematical practices such as making sense of quantities (**MHM2**), calculating accurately (**MHM6**), and making use of number patterns and structures in the number system (**MHM7**).

Example: Find the sum of 43 + 34 + 57 + 24	M.2.10
Student A (Commutative and Associative Properties). "I saw the 43 and 57 and added them first. I know 3 plus 7 equals 10, so when I added them, 100 was my answer. Then I added 34 and had 134. Then I added 24 and had 158. So, 43 + 57 + 34 + 24 = 158."	
Student B (Place Value Strategies). "I broke up all of the numbers into tens and ones. First I added the tens: 40 + 30 + 50 + 20 = 140. Then I added the ones: 3 + 4 + 7 + 4 =18. That meant I had 1 ten and 8 ones. So, 140 + 10 is 150. 150 and 8 more is 158. So, 43 + 34 + 57 + 24 = 158."	
Student C (Place Value Strategies and Commutative and Associative Property). "I broke up all the numbers into tens and ones. First I added up the tens: 40 + 30 + 50 + 20. I changed the order of the numbers to make adding easier. I know that 30 plus 20 equals 50, and 50 more equals 100. Then I added the 40 and got 140. Then I added up the ones: 3 + 4 + 7 + 4. I changed the order of the numbers to make adding easier. I know that 3 plus 7 equals 10 and 4 plus 4 equals 8. I also know that 10 plus 8 equals 18. I then combined my tens and my ones: 140 plus 18 (1 ten and 8 ones) equals 158."	

Adapted from NCDPI 2013b.

Finally, students explain why addition and subtraction strategies work, using place value and the properties of operations (**M.2.13**). Grade two students need multiple opportunities to explain their addition and subtraction thinking (**MHM2**). For example, students use place value understanding, properties of operations, number names, words (including mathematical language), math drawings, number lines, and physical objects to explain why and how they solve a problem (**MHM1, MHM6**). Students can also critique the work of other students (**MHM3**) to deepen their understanding of addition and subtraction strategies.

Example	**M.2.13**
There are 36 birds in the park. Suddenly, 25 more birds arrive. How many birds are there? Solve the problem and show your work. **Student A.** "I broke 36 and 25 into tens and ones (30 + 6) + (20 + 5). I can change the order of my numbers, since it doesn't change any amounts, so I added 30 + 20 and got 50. Then I added 5 and 5 to make 10 and added it to the 50. So, 50 and 10 more is 60. I added the one that was left over and got 61. So, there are 61 birds in the park."	
Student B. "I used a math drawing and made a pile of 36 and a pile of 25. Altogether, I had 5 tens and 11 ones. 11 ones is the same as one ten and one left over. So, I really had 6 tens and 1 one. That makes 61."	

Adapted from NCDPI 2013b.

Instructional Focus
When students explain why addition and subtraction strategies work (**M.2.13**), they reinforce foundations for solving one- and two-step word problems (**M.2.1**) and extend their understanding and use of various strategies and models, drawings, and a written method to add and subtract (**M.2.9** and **M.2.11**).

Students are to fluently add and subtract within 100 in grade two (using place value strategies, properties of operations, and/or the relationship between addition and subtraction) (**M.2.9**). In grade one, students add within 100 using concrete models or drawings and use at least one method that is generalizable to larger numbers (such as between 101 and 1000). In grade two, students extend addition to within 1000 using these generalizable concrete methods. This extension could be connected first to adding all two-digit numbers (e.g., 78 + 47) so that students can see and discuss composing both ones and tens without the complexity of hundreds in the drawings or numbers.

After solving addition problems that compose both ones and tens for all two-digit numbers and then three-digit numbers within 1000, the fluency problems for grade two seem easy:

28 + 47 requires composing only the ones. This is now easier to do without drawings: one just records the new ten before it is added in, or adds it in mentally. Fluent adding means adding without drawings.

The same approach may be taken for subtraction, first solving with concrete models or drawings of subtractions within 100 that involve decomposing 1 ten to make 10 ones and then solving subtraction problems that require two decompositions, of 1 hundred to make 10 tens and of 1 ten to make 10 ones. Spending a long time subtracting within 100 initially can stimulate students to count on or count down, methods that become considerably more difficult above 100. Problems with all possibilities of decompositions are mixed in so that students solve problems requiring two, one, and no decompositions. Then students can spend time on subtractions that include multiple hundreds (totals from 201 to 1000). After this experience, focusing within 100 just on the two cases of one decomposition (e.g., 73 – 28) or no decomposition (e.g., 78 – 23) is relatively easy to do without drawings.

Number talks as an instructional tool.
Many teachers incorporate an activity known as " number talks" into their classrooms. The teacher typically writes a problem on the board (such as 45 + 47) and asks students to solve the problem only through a mental process. The teacher then records all answers given by students, whether correct or incorrect, without judgment. A class discussion follows; students explain how they got their answers and decide which answer is correct. The class may agree or disagree with a particular method, find out where another student made an error, or compare different solution methods (e.g., how finding 45 + 45 + 2 is similar to finding 40 + 40 + 12). In number talks, multiple strategies often emerge naturally from the students, and opportunities to explore these strategies arise. When students do not have more than one strategy for solving a problem, this can be an indication to the teacher that students have a limited repertoire of such strategies, and therefore number talks can be used as a valuable instructional tool. Number talks support several Mathematical Habits of Mind, including **MHM1**, **MHM2**, **MHM3**, **MHM7**, and **MHM8**. (Standard **M.2.12** calls for grade two students to practice mental math by adding and subtracting multiples of 10 and 100 from a given number between 100 and 900.)

Domain: Measurement and Data

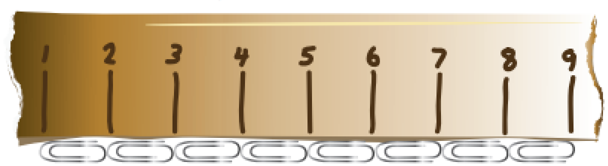
Grade two students transition from measuring lengths with informal or non-standard units to measuring with standard units—inches, feet, centimeters, and meters—and using standard measurement tools (**M.2.14**). Students learn the measure of length as a count of how many units are needed to match the length of the object or distance being measured. Using both customary units (inches and feet) and metric units (centimeters and meters), students measure the length of objects with rulers, yardsticks, meter sticks, and tape measures. Students become familiar with standard units (e.g., 12 inches in a foot, 3 feet in a yard, and 100 centimeters in a meter) and how to estimate lengths (adapted from KATM 2012, 2nd Grade Flipbook).

Measurement and Data
Measure and estimate lengths in standard units.
M.2.14 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
M.2.15 Measure the length of an object twice, using length units of different lengths for the two measurements, describe how the two measurements relate to the size of the unit chosen.
M.2.16 Estimate lengths using units of inches, feet, centimeters, and meters.
M.2.17 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Students also can learn accurate measurement procedures and concepts by constructing simple unit rulers (see the adjacent figure). Using copies of a standard unit, such as manipulatives that measure one inch, students mark off unit lengths on strips of paper, explicitly connecting the process of measuring with a ruler to measuring by iterating physical units.

Figure 2-5. Example of a Student-Created Ruler

Using a unit to draw a ruler



Students use a standard unit (shown below the ruler) to make rulers, helping them to understand the meaning of the marks on rulers.

Adapted from UA Progressions Documents 2012a.

Thus, students' first rulers are simple tools to help count the iteration of unit lengths. Frequently comparing results of measuring the same object with manipulatives of standard unit length (e.g., a block that is one inch long) and with student-created rulers can help students connect their experiences and ideas. As they build and use these tools, they develop the ideas of unit length iteration (unit lengths are all of equal size), correct alignment (with a ruler), measurement of the length between hash marks on the ruler, and the zero-point concept (the idea that the zero of the ruler indicates one endpoint of a length).

Grade two students learn the concept of the inverse relationship between the size of the unit of length and the number of units required to cover a definite length or distance—specifically, that the larger the unit, the fewer units are needed to measure something, and vice versa (**M.2.15**). Students measure the length of the same object using units of different lengths (ruler with inches versus ruler with centimeters, or a foot ruler versus a yardstick) and discuss the relationship between the size of the units and the measurements.

Example	**M.2.15**
A student measured the length of a desk in both feet and inches. The student found that the desk was 3 feet long and that it was 36 inches long. **Teacher:** "Why do you think you have two different measurements for the same desk?" **Student:** "It only took 3 feet because the feet are so big. It took 36 inches because an inch is much smaller than a foot."	

Students use this information to understand how to select appropriate tools for measuring a given object. For instance, a student might think, "The longer the unit, the fewer units I need." Measurement problems also support mathematical practices such as reasoning quantitatively (**MHM2**), justifying conclusions (**MHM3**), using appropriate tools (**MHM5**), attending to precision (MHM6), and making use of structure or patterns (**MHM7**).

After gaining experience with measurement, students learn to estimate lengths using units of inches, feet, centimeters, and meters (**M.2.16**). Students estimate lengths before they measure. After measuring an object, students discuss their estimations, measurement procedures, and the differences between their estimates and the measurements. Students begin by estimating measurements of familiar items (e.g., the length of a desk, pencil, favorite book, and so forth). Estimation helps students focus on the attribute to be measured, the length units, and the process. Students need many experiences with the use of measurement tools to develop their understanding of linear measurement; an example is provided below.

Example	**M.2.16**
Teacher: "How many inches do you think this string is if you measure it with a ruler?" **Student:** "An inch is pretty small. I'm thinking it will be somewhere between 8 and 9 inches." **Teacher:** "Measure it and see." **Student:** "It is 9 inches. I thought that it would be somewhere around there."	

This example also supports mathematical practices such as making sense of quantities (**MHM2**) and using appropriate tools strategically (**MHM5**).

Students also measure to determine the difference in length between one object and another, expressing the difference in terms of a standard length unit (**M.2.17**). Grade two students use inches, feet, yards, centimeters, and meters to compare the lengths of two objects. They use comparative phrases such as "It is 2 inches longer" or "It is shorter by 5 centimeters" to describe the difference in length between the two objects. Students use both the quantity and the unit name to precisely compare length (ADE 2010 and NCDPI 2013b).

Instructional Focus
As students compare objects by their lengths, they also reinforce skills and understanding related to solving “compare” problems in the cluster “Represent and solve problems involving addition and subtraction.” Drawing comparison bars to represent the different measurements helps make this link explicit (see standard **M.2.1**).

Students apply the concept of length to solve addition and subtraction problems. Word problems refer to the same unit of measure (**M.2.18**).

Measurement and Data

Relate addition and subtraction to length.

M.2.18

Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units (e.g., by using drawings, such as drawings of rulers), and equations with a symbol for the unknown number to represent the problem.

M.2.19

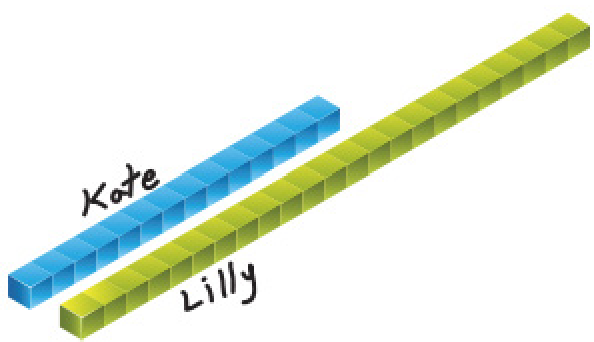
Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2… and represent whole-number sums and differences within 100 on a number line diagram.

In grade two, students also connect the concept of the ruler to the concept of the number line. These understandings are essential to supporting work with number line diagrams.

Example	**M.2.18**
Kate jumped 14 inches in gym class. Lilly jumped 23 inches. How much farther did Lilly jump than Kate? Solve the problem and then write an equation.	

Student A: My equation is 14 + __ = 23. I thought, “14 and what makes 23?” I used cubes. I made a train of 14. Then I made a train of 23. When I put them side by side, I saw that Kate would need 9 more cubes to be the same as Lilly. So, Lilly jumped 9 more inches than Kate.

14 + 9 = 23. (**MHM1, MHM2, MHM4**)



Student B: My equation is 23 – 14 = __. I thought about what the difference was between Kate and Lilly. I broke up 14 into 10 and 4. I know that 23 minus 10 is 13. Then, I broke up the 4 into 3 and 1. 13 minus 3 is 10. Then, I took one more away. That left me with 9. So, Lilly jumped 9 inches more than Kate. That seems to make sense, since 23 is almost 10 more than 14.

23 – 14 = 9. (**MHM2, MHM7, MHM8**)

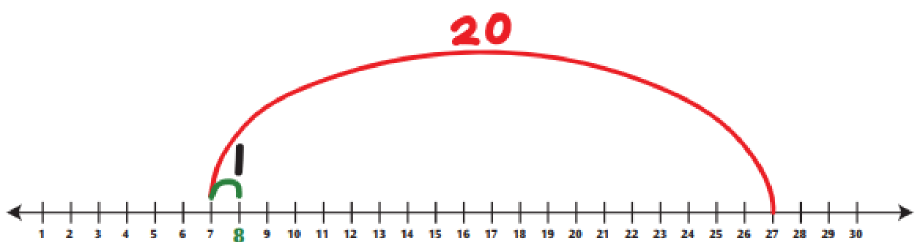
Instructional Focus

Addition and subtraction word problems involving lengths develop mathematical practices such as making sense of problems (**MHM1**), reasoning quantitatively (**MHM2**), justifying conclusions (**MHM3**), using appropriate tools strategically (**MHM5**), attending to precision (**MHM6**), and evaluating the reasonableness of results (**MHM8**). Similar word problems also support students' ability to fluently add and subtract, which is part of the major work at the grade (refer to fluency expectations in standards **M.2.1** and **M.2.18**).

Using a number line diagram to understand number and number operations requires students to comprehend that number line diagrams have specific conventions: namely, that a single position is used to represent a whole number and that marks are used to indicate those positions. Students need to understand that a number line diagram is like a ruler in that consecutive whole numbers are one unit apart; thus, they need to consider the distances between positions and segments when identifying missing numbers. These understandings underlie the successful use of number line diagrams. Students think of a number line diagram as a measurement model and use strategies relating to distance, proximity of numbers, and reference points (UA Progressions Documents 2012a).

Example **M.2.19**

There were 27 students on the bus. Nineteen (19) students got off the bus. How many students are on the bus?
Student: I used a number line. I saw that 19 is really close to 20. Since 20 is a lot easier to work with, I took a jump of 20. But, that was one too many. So, I took a jump of 1 to make up for the extra. I landed on 8. So, there are 8.



Adapted from ADE 2010 and NCDPI 2013b.

Teachers ensure that students make the connection between problems involving measuring with a ruler and those involving a number line as a problem-solving strategy.

Instructional Focus

Using addition and subtraction within 100 to solve word problems involving length (**M.2.18**) and representing sums and differences on a number line (**M.2.19**) reinforces the use of models to add and subtract and supports major work at grade two (see standards **M.2.1** and **M.2.11**). Similar problems also develop mathematical practices such as making sense of problems (**MHM2**), justifying conclusions (**MHM3**), and modeling mathematics (**MHM4**).

In grade one, students learn to tell time to the nearest hour and half-hour. Grade two students tell time to the nearest five minutes (**M.2.20**).

Measurement and Data
Relate addition and subtraction to length.
M.2.20
Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
M.2.21
Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately (e.g., If you have 2 dimes and 3 pennies, how many cents do you have?).

Students can make connections between skip-counting by fives (**M.2.6**) and five-minute intervals on the clock. Students work with both digital and analog clocks. They recognize time in both formats and communicate their understanding of time using both numbers and language.

Grade two students also understand that there are two 12-hour cycles in a day—a.m. and p.m. A daily journal can help students make real-world connections and understand the differences between these two cycles.

Instructional Focus
Students' understanding and use of skip-counting by fives and tens (**M.2.6**) can also support telling and writing time to the nearest five minutes (**M.2.20**). Students notice the pattern of numbers and apply this understanding to time (**MHM7**).

In grade two, students solve word problems involving dollars or cents (**M.2.21**). They identify, count, recognize, and use coins and bills in and out of context. Grade two students need opportunities to make equivalent amounts using both coins and bills. Dollar bills include denominations up to one hundred ($1, $5, $10, $20, $50, $100). Note that grade two students do not express money amounts using decimal points.

Just as students learn that a number may be represented in different ways and still remain the same amount — e.g., 38 can be 3 tens and 8 ones or 2 tens and 18 ones — students can apply this understanding to money. For example, 25 cents could be represented as a quarter, two dimes and a nickel, or 25 pennies, all of which have the same value. Building the concept of equivalent worth takes time, and students will need numerous opportunities to create and count different sets of coins and to recognize the "purchasing power" of coins (e.g., a girl can buy the same things with a nickel that she can purchase with 5 pennies).

As teachers provide students with opportunities to explore coin values (e.g., 25 cents), actual coins (e.g., 2 dimes and 1 nickel), and drawings of circles that have values indicated, students gradually learn to mentally assign a value to each coin in a set, place a random set of coins in order, use mental math, add on to find differences, and skip-count to determine the total amount.

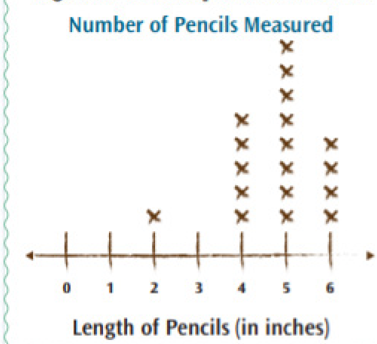
Example	M.2.21
Using pennies, nickels, dimes, and quarters, how many different ways can you make 37 cents?	
Using $1, $5, and $10 bills, how many different ways can you make $12?	

Adapted from ADE 2010 and NCDPI 2013b.

Measurement and Data
Relate addition and subtraction to length. **M.2.22** Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **M.2.23** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Students use the measurement skills described in previous standards (**M.2.14–17**) to measure objects and create measurement data (**M.2.22**). For example, they measure objects in their desk to the nearest inch, display the data collected on a line plot, and answer related questions. Line plots are first introduced in grade two. A line plot can be thought of as plotting data on a number line (see figure below).

Figure 2-6. Example of a Line Plot



In grade one, students work with three categories of data. In grade two, students work with data that have up to four categories. Students organize and represent data on a picture graph or bar graph (with single-unit scale) and interpret the results. They solve simple put-together, take-apart, and “compare” problems using information presented in a bar graph (**M.2.23**). In grade two, picture graphs (pictographs) include symbols that represent single units. Pictographs should include a title, categories, category label, key, and data.

Instructional Focus
Students use data to pose and solve simple one-step addition and subtraction problems. The use of picture graphs and bar graphs to represent a data set (**M.2.23**) reinforces grade level work in the cluster “Represent and solve problems involving addition and subtraction” and provides a context for students to solve related addition and subtraction problems (**M.2.1**).

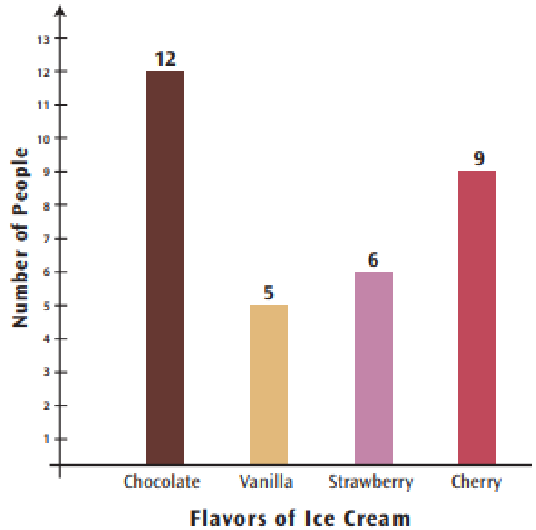
Example **M.2.23**

Students are responsible for purchasing ice cream for an event at school. They decide to collect data to determine which flavors to buy for the event. Students decide on the question to ask their peers—"What is your favorite flavor of ice cream?"— and four likely responses: chocolate, vanilla, strawberry, and cherry. Students form two teams and collect information from different classes in their school. Each team decides how to keep track of its data (e.g., with tally marks, check marks, or in a table). Each team selects either a picture graph or a bar graph to display its data. Graphs are created using paper or a computer.

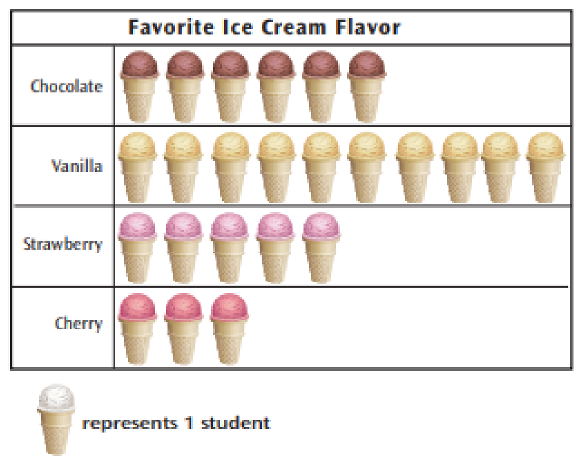
The teacher facilitates a discussion about the data collected, asking questions such as these:

- Based on the graph from Team A, how many students voted for cherry, strawberry, vanilla, or chocolate ice cream?
- Based on the graph from Team B, how many students voted for cherry, strawberry, vanilla, or chocolate ice cream?
- Based on the data from both teams, which flavor received the most votes? Which flavor received the fewest votes?
- What was the second-favorite flavor?
- Based on the data collected, what flavors of ice cream do you think we should purchase for our event, and why do you think that?

Team A: Bar Graph



Team B: Picture Graph



Representing and interpreting data to solve problems also develops mathematical practices such as making sense of problems (**MHM1**), reasoning quantitatively (**MHM2**), justifying conclusions (**MHM3**), using appropriate tools strategically (**MHM5**), attending to precision (**MHM6**), and evaluating the reasonableness of results (**MHM8**).

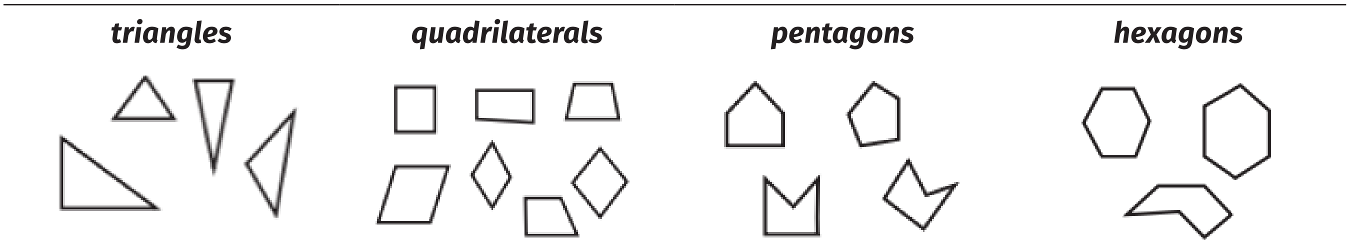
Domain: Geometry

A critical area of instruction in grade two is for students to describe and analyze shapes by examining their sides and angles. This work develops a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Geometry
Reason with shapes and their attributes. **M.2.24** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (sizes are compared directly or visually, not compared by measuring). Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **M.2.25** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **M.2.26** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Students identify, describe, and draw triangles, quadrilaterals (squares, rectangles and parallelograms, and trapezoids), pentagons, hexagons, and cubes (**M.2.24**); see the examples below. Pentagons, triangles, and hexagons appear as both regular (having equal sides and equal angles) and irregular. Second-grade students recognize all four-sided shapes as quadrilaterals. They use the vocabulary word angle in place of corner, but they do not need to name angle types (e.g., right, acute, obtuse). Shapes are presented in a variety of orientations and configurations.

Examples of the Presentation of Various Shapes



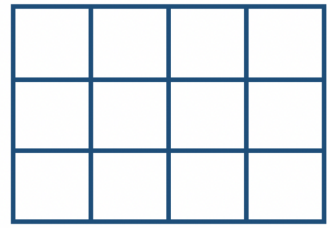
As students use attributes to identify and describe shapes, they also develop mathematical practices such as analyzing givens and constraints (**MHM1**), justifying conclusions (**MHM3**), modeling with mathematics (**MHM4**), using appropriate tools strategically (**MHM5**), attending to precision (**MHM6**), and looking for a pattern or structure (**MHM7**).

Students partition a rectangle into rows and columns of same-size squares and count to find the total number of squares (**M.2.25**). As students partition rectangles into rows and columns, they build a foundation for learning about the area of a rectangle and using arrays for multiplication.

Example **M.2.25**

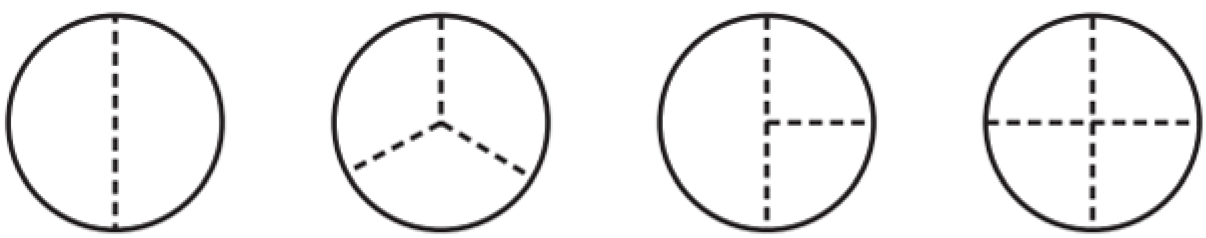
Teacher: Partition this rectangle into 3 equal rows and 4 equal columns. How can you partition into 3 equal rows? Then into 4 equal columns? Can you do it in the other order? How many small squares did you make?

Student: I counted 12 squares in the rectangle. This is a lot like when we counted arrays by counting 4 + 4 + 4 = 12.



An interactive whiteboard or manipulatives such as square tiles, cubes, or other square-shaped objects can be used to help students partition rectangles (**MHM5**).

In grade one, students partitioned shapes into two and four equal shares, describing the shares using words halves, fourths, and quarters. Grade two students partition circles and rectangles into two, three, or four equal shares (regions). Students explore this concept with paper strips and pictorial representations and work with the vocabulary terms halves, thirds, and fourths (**M.2.26**). Students recognize that when they cut a circle into three equal pieces, each piece will equal one-third of its original whole and the whole may be described as three-thirds. If a circle is cut into four equal pieces, each piece will equal one-fourth of its original whole, and the whole is described as four-fourths.



Circle cut into halves | Circle cut into thirds | Circle *not* cut into thirds | Circle cut into fourths

Students see circles and rectangles partitioned in multiple ways so they learn to recognize that equal shares can be different shapes within the same whole.

As students partition circles and squares and explain their thinking, they develop mathematical practices such as making sense of quantities (**MHM2**), justifying conclusions (**MHM3**), attending to precision (**MHM6**), and evaluating the reasonableness of their results (**MHM7**). They also develop understandings that will support grade three work in the cluster “Develop understanding of fractions as numbers” (**M.3.13–M.3.15**) [adapted from ADE 2010 and NCDPI 2013b].

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, when done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.

In kindergarten through grade two, students focus on addition, subtraction, and measurement using whole numbers. To be prepared for grade three mathematics, students are able to demonstrate by the end of grade two that they have acquired specific mathematical concepts and procedural skills and have met the fluency expectations for the grade. For grade two students, the expected fluencies are to add and subtract within 20 using mental strategies. They know from memory all sums of two one-digit numbers (**M.2.2**), and to add and subtract within 100 using various strategies (**M.2.9**). These fluencies and the conceptual understandings that support them are foundational for multiplicative reasoning and fractional understanding in later grades.

Of particular importance at grade two are concepts, skills, and understandings of:

- addition and subtraction within 20 and representing and solving problems involving addition and subtraction (**M.2.1–2**)
- place value (**M.2.5–8**) and the use of place value understanding and properties of operations to add and subtract (**M.2.9–13**)
- measuring and estimating lengths in standard units (**M.2.14–17**)
- relating addition and subtraction to length (**M.2.18–19**).

Place Value

By the end of grade two, students are expected to read, write, and count to 1000, skip-counting by twos, fives, tens, and hundreds. Students need to understand that 100 can be thought of as a bundle of 10 tens and also understand three-digit whole numbers in terms of hundreds, tens, and ones.

Addition and Subtraction

Addition and subtraction are major instructional focuses in kindergarten through grade two. By the end of grade two, students are expected to add and subtract (using concrete models, drawings, and strategies) within 1000 (**M.2.11**). Students add and subtract fluently within 100 using various strategies (**M.2.9**), and add and subtract fluently within 20 using mental strategies (**M.2.2**). Students mentally add and subtract 10 or 100, within the range 100–900 (**M.2.12**). They are expected to know from memory all sums of two one-digit numbers (**M.2.2**). Students also know how to apply addition and subtraction to solve a variety of one- and two-step word problems (within 100) involving add-to, take-from, put-together, take-apart, and compare situations (**M.2.1**).

Students who have met the grade two standards for addition and subtraction will be prepared to fluently add and subtract within 1000 using strategies and algorithms, as required in the grade three standards. These foundations will also prepare students for concepts, skills, and problem solving with multiplication and division, which are introduced in grade three.

Measurement

By the end of grade two, students can measure lengths using standard units — inches, feet, centimeters, and meters. Students need to know how to use addition and subtraction within 100 to solve word problems involving lengths (**M.2.18**). Mastering grade two measurement standards will prepare students to measure fractional amounts and to add, subtract, multiply, or divide to solve word problems involving mass or volume, as required in the grade three standards.

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