WEST VIRGINIA
DEPARTMENT OF EDUCATION

MATHEMATICS

GRADE 9

Bagel Shop

Table of Contents

Task Title: Functions: Describing and Graphing Situations: Bagel Shop
Grade or Content Area: 9th Grade
Toolkit Author: Diane Furman, Diana Munza, and Melissa Lough
Original Task Creator: Illustrative Mathematics
Quarter: 1

Rationale for Lesson and Associated Tasks

Functions are essential in every field of applied mathematics. Mathematicians use the idea of a *function* to describe operations such as addition and multiplication, transformations of geometric figures, relationships between variables, and many other things. They are useful to the statistician who uses functions to tell if a certain outlying observation is significant or expected. The climatologist has a single-valued mapping of a given year to the global mean temperature of that year. Government accountants have single-valued mappings of a given year to the amount of revenue that the government should expect to take in under current tax policy. If the climatologist's or the accountant's model for temperature or revenue returned more than one value for a given year, it would not be a function—and it would not be very useful.

In grade 8, students learned that a function is a rule that assigns exactly one output to each input. They represented functions in different ways—with verbal descriptions, algebraic expressions, graphs, and tables—and used functions to model relationships between quantities, linear relationships in particular. In this lesson, students build on what they learned about functions in grade 8. Students develop their capacity to represent, interpret, and use functions to make sense of quantities in situations and to solve problems.

Lesson and Associated Tasks Overview

Illustrative Mathematics (click here)
(*Review all components of the task thoroughly*)

The lesson and associated *Bagel Shop* task serve as an introductory activity to the study of Functions and their representations. Students consider why certain relationships could be functions while others could not be. By analyzing tables and graphs that represent both functions and non-functions, and by interpreting a description of each situation, students are reminded that a function assigns exactly one output value to each input. If an input could have more than one possible output, then the relationship cannot be a function.

In the study of Functions and Their Representations, students expand and deepen their understanding of functions. They develop new knowledge and skills for communicating about functions clearly and precisely, investigate different kinds of functions, and hone their ability to interpret functions. Students also use functions to model a mathematical and real-world situation.

This lesson and associated tasks are scheduled to be completed over 2 class periods per the suggested sequence:

Day 1

1. Introduce Learning Target for the lesson
2. Distribute Student Task Statement for Lesson 1: Describing and Graphing Situations

3. Introduce, play, and discuss the YouTube video, *Algebra Basics: What Are Functions?* (click here)
4. Engage students in an exploration of an input-output function with *Bagel Shop* (Task 1.1)
5. Student Journal prompt: If it costs $3 per hour to park in a parking lot, with a maximum cost of $12; explain why the amount of time a car is parked is *not* a function of the parking cost.

Day 2

1. Review Learning Target and activities from Day 1
2. Introduce Learning Targets for the lesson
3. Engage students in reasoning graphically about functions with *Be Right Back!* (Task 1.2)
4. Introduce vocabulary of functions through *Talk about a Function* (Task 1.3)
5. Lesson Synthesis: Whole-class discussion
6. Student Journal: Check for Understanding - Respond to questions - *The Backyard Pool* (Task 1.4)

West Virginia College-and Career-Readiness State Standards (Integrated)

M.1HS.12

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation *y = f(x).* Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.

M.1HS.15

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions

West Virginia College- and Career-Readiness State Standards (Traditional)

M.A1HS.18

Recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x).* Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains.

M.A1HS.21

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.

Mathematical Habits of Mind (MHM)

MHM1. Make sense of problems and persevere in solving them.
MHM4. Model with mathematics.

Mathematics Teaching Practices to Support Student Growth

- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Support productive struggle in learning mathematics**.**

Essential Understandings

- A relationship between two quantities is a function if there is only one possible output for each input.
- Words, tables, and graphs can represent relationships that are functions, including identifying the independent and dependent variables.

Set-up Phase

1. Become an Expert Regarding All Lesson and Associated Tasks Content

It is imperative that the teacher becomes very familiar with all *Function and Their Representations: Describing and Graphing Situations* materials (e.g. Teacher Guide, Teacher Presentation Materials, Student Task Statements, Blackline Master, and YouTube video). Each lesson has several associated tasks. The implementation of the tasks may vary class to class depending on the time frame available for the lessons, and the levels of student engagement and understanding. Suggestions for *Students with Disabilities* and *English Language Learners* are provided within the Unit.

2. Establish Small Groups

Functions: Describing and Graphing Situations tasks and activities engage students in both individual and group thinking. Working individually, in pairs and in small groups, students will explore and share their reasoning about functions and their representations.

Small group instruction has significant impact on student achievement (Hattie, 2009) and allows teachers to work more closely with each student. This type of instruction provides the opportunity to monitor students' learning and identify gaps in the development of students' math skills.

Teachers will need to instruct students on how to work in small groups. The first step in the process is to establish ground rules and norms for interaction. Students must have a part in making the rules and these guidelines must be enforced by both teachers and students. Ground rules should encourage positive collaborative behaviors among all students. Guidelines/ground rules need to be posted in the classroom so students can readily refer to them. If students or teachers believe that additional rules are needed, they can be added later.

Teachers should intentionally assign groups based on skills and/or backgrounds. Skill levels, leadership skills, and personalities must all be considered when creating small groups. Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis. Observations of leadership skills; personalities; ability to take criticism; ability to question; and the ability to think deeply about a task or problem will prove to be extremely helpful when creating small groups for this unit. This strategy minimizes the chance that high ability students will flock

together leaving others out, allows teachers to create more diverse groups, and creates opportunities for students to work with peers they otherwise might not have interacted.
Small group instruction gives teachers an opportunity to assess more closely what each student can do and build strategic plans around those assessments. Students who struggle to ask questions and participate in a whole group setting may thrive in a small group where they feel more comfortable and less overwhelmed. Furthermore, small group instruction tends to proceed at a fast pace, which typically helps students maintain focus and enjoy the learning experience.
Source: *Small Group Instruction: How to Make it Effective*, CORE: Excellence in Education Blog, September 27, 2018.

3. Develop Open-Ended Questions

Teachers of *Describing and Graphing Situations* should use open-ended questions to support and scaffold the lessons and associated tasks for their students. These questions should purposefully direct students towards provided information, previously learned content, and similarities and differences in their work versus other group members. Within the context of open-ended mathematical tasks, teachers should select questions with four purposes in mind:

A. *Starter questions*

Starter questions focus the students' thinking in a general direction to give a starting point, including:

- How could you sort these.....?
- How many ways can you find to .....?
- What happens when we .....?
- What can be made from .....?
- How many different ..... can be found?

B. *Questions to stimulate mathematical thinking*

These questions focus on particular strategies aiding the formation of a strong conceptual network. The questions serve as a prompt when students become 'stuck' and examples include:

- What is the same?
- What is different?
- Can you group these ..... in some way?
- Can you see a pattern?
- How can this pattern help you find an answer?
- What do you think comes next? Why?
- Is there a way to record what you have found that might help us see more patterns?

C. *Assessment questions*

Assessment questions ask students to explain what they are doing or how they arrived at a solution. They allow the teacher to see how students are thinking, what students understand, and at what level students are operating mathematically. These open-ended questions are best asked after time has been given to make progress with the problem, to record some findings, and perhaps to have achieved at least one solution. Assessment questions might include:

- What have you discovered?
- How did you find that out?
- Why do you think that?
- What made you decide to do it that way?

D. *Final discussion questions*

Final discussion questions draw together the efforts of the class and prompt sharing and comparison of strategies and solutions. This is a vital phase in the mathematical thinking processes. It provides further opportunity for reflection and realization of mathematical ideas and relationships. It encourages students to evaluate their work. Final discussion questions might include:

- Who has the same answer/ pattern/ grouping as this?
- Who has a different solution?
- Are every one's results the same?
- Why/why not?
- Have we found all the possibilities?
- How do we know?
- Have you thought of another way this could be done?
- Do you think we have found the best solution?

*Additional questions are provided in the Teacher Guide.

Source: *Using Questioning to Stimulate Mathematical Thinking*, NRICH, February 2011.

4. Gather Materials

- Computer and presentation device(s)
- Internet access (test prior to implementation)
- Graphing calculators
- Video (Online) *Algebra Basics: What Are Functions?* by Math Antics (11:33) https://www.youtube.com/watch?v=52tpYl2tTqk
- Illustrative Mathematics (Algebra Unit 4, Lesson 1) Student Task Statement handouts *Bagel Shop* (Unit 4, Lesson 1, Task 1.1); *Be Right Back!* (Unit 4, Lesson 1, Task 1.2); *Talk About a Function* (Unit 4, Lesson 1, Task 1.3); and *The Backyard Pool* (Unit 1, Lesson 1, Task 1.4) *https://curriculum.illustrativemathematics.org/HS/students/1/4/1/index.html*
- Unit 4 Lesson 1 *Describing and Graphing Situations* Student Task Statements
- Student Math Journals

5. Anticipated Common Student Misconceptions

- Students may think that all functions can be written as equations.
- Students may think that the range must map back onto the domain with single values.
- Students may misunderstand how restrictions on the domain of a relationship affect the relationship.
- Students often confuse when they are looking for the input value and when they are looking for the output value

Explore Phase

In general, students understand that functions describe situations in which one quantity determines another. The main work in grade eight concerns linear functions, though students are exposed to non-linear functions to contrast them with linear functions. Thus, students may view a linear equation such as $y = -0.75x + 12$ as a rule that defines a quantity y whenever the quantity x is given. In this case, the function may describe the amount of money remaining after a number of turns (x), when a student starts with \$12 plays a game that costs \$0.75 per turn. Alternatively, students may view the formula for the area of a circle, $A = \pi r^2$ as a (non-linear) function in the sense that the area of a circle is dependent

on its radius. Student work with functions at grade eight remains informal. Therefore, students will need ample opportunities (e.g. "bell ringers") to explore these three standards prior to the introduction of the *Functions: Describing and Graphing Situations* lessons and tasks:

- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- Construct a function to model a linear relationship between two quantities.
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).

Prior Instruction/Knowledge

The study of proportional relationships in grade seven is a foundation for the study of functions that is introduced in grade eight and continues through higher mathematics. In grade eight, students understand that the proportional relationships they studied in grade seven are part of a broader group of linear functions. Linear functions are characterized by having a constant rate of change (the change in the outputs is a constant multiple of the change in the corresponding inputs).

Please review the following:

Educators Guide for Mathematics: Grade 7 (pages 6-7, pdf 8-9) click here
Educators Guide for Mathematics: Grade 8 (pages 19-23, pdf 21-25) click here

Prerequisite Skills

- Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form. (930Q)
- Locate points in all quadrants of the coordinate plane using ordered pairs in number and word problems.

Supporting Skill

- Given a list of ordered pairs in a table or graph identify either verbally or algebraically the rule used to generate and record the results.

Impending Skills

- Use and interpret function notation in number and word problems; determine a value of the function given an element of the domain.
- Determine compositions of functions.
- Graph sine and cosine functions and identify the domain, range, period, amplitude, midline, and phase shift of the function.
- Graph a radical relation, function, or inequality. State the domain and range.
- Write the equation of and graph exponential equations or functions, including $f(x) = ab^x$ and $f(x) = a(1+r)^x$, in number and word problems; identify and interpret critical values.
- Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables and any constraints of the domain or range.

Source: *The Quantile Framework for Mathematics* 2020 MetaMetrics Inc

Implementation Phase

Day 1

1. Introduce Learning Target for the lesson
2. Distribute Student Task Statement for Lesson 1: Describing and Graphing Situations
3. Introduce, play, and discuss the YouTube video, *Algebra Basics: What Are Functions?* (click here)
4. Engage students in an exploration of an input-output function with *Bagel Shop* (Task 1.1)
5. Student Journal prompt: If it costs $3 per hour to park in a parking lot, with a maximum cost of $12; explain why the amount of time a car is parked is *not* a function of the parking cost.

Day 1 Teacher Notes:

To introduce the lesson, briefly explain the Learning Target:

- Explain when a relationship between two quantities is a function

Distribute graphing calculators and Student Task Statements:

- Algebra Unit 4, Lesson 1 *Bagel Shop* (Unit 4, Lesson 1, Task 1.1)
- Blackline Master (Unit 4, Lesson 1, Task 1.1*Bagel Shop)*

1. Arrange students in groups of four.
2. Introduce the YouTube video, *Algebra Basics: What are Functions* by asking the students (whole-class) what they remember about functions from grade 8. Record all student responses and display for all students to see.
3 Play the video.
4. Discuss in whole-class setting what the students learned from the video. Record the responses. Compare with the previous list. Was there new information in the video? Did they correctly remember information about functions from grade 8?
5. Ask students to read the first question on the Student Statement Task handout, 1.1 *Bagel Shop*. Give students a few minutes to think about the first question. Have students to share their thinking with group members.
6. Engage students in a whole-class discussion: Can the shopkeeper and customers in the Bagel Shop all be right about the cost of 13 bagels? If so, how? Record and display student responses.
7. Ask students to look at the Blackline Master Task 1.1 *Bagel Shop*. Direct students to complete the table to show how each person in the Bagel Shop calculated the total price of 13 bagels. Tell students to leave the Best Price column blank for now. Inform students they may use their graphing calculators for the task and they may work together on the task in their groups.
8. Circulate the room listening to students as you allow students enough time to find the answers.
9. Based on your observations, select 2 groups to model and share their findings with the whole class.
10. Ask whole-class, if we graph the relationship between Number of Bagels and Price, what do you think the graph will look like?
11. Ask students to graph the 4 columns of data (Numbers of Bagels and Price) on their calculators. Inform the students to make each set of data a different color (e.g., Shopkeeper - solid blue dots; Jada – open green circles; Prya – red squares and Han – yellow triangles.)
12. Discuss with the whole class if the graph confirms that the Number of Bagels and Price is a function or not. Why?
13. Direct students to work in their groups to compare the prices of 1 through 13 bagels for each character. Tell students to complete the Best Price Column of the Blackline Master

handout. Once they have completed the column, ask students to graph on their calculators the data for Numbers of Bagels and Best Price.

14. Circulate the room listening to students as you allow students time to find the answers.
15. Discuss in whole-class how the Numbers of Bagels and Best Price graph is different from the first graph. Does the second graph confirm that the Number of Bagels and Best Price is a function or not. Why? What defines a function?
16. Provide the following prompt for a Student Journal entry: If it costs $3 per hour to park in a parking lot, with a maximum cost of $12, explain why the amount of time a car is parked is *not* a function of the parking cost.

Day 2

1. Review Learning Target and activities from Day 1
2. Introduce Learning Targets for the lesson
3. Engage students in reasoning graphically about functions with *Be Right Back!* (Task 1.2)
4. Introduce vocabulary of functions through *Talk about a Function* (Task 1.3)
5. Lesson Synthesis: Whole-class discussion
6. Student Journal - Check for Understanding: Respond to questions Unit 4 Lesson 1 Cool Down Task 1.4 *The Backyard Pool*

Day 2 Teacher Notes:

Review the Day 1 Learning Target by having students share their journal entries with members of their group. Possible Entry: If the amount of parking time is a function of the parking cost, then the cost is the input and the amount of parking time is the output. When the parking cost (input) is $12, the amount of parking time (output) could be 4, 6, 7.5, 11, or any length that is at least 4 hours. A relationship is not a function if for each input there are multiple possible outputs.

To introduce the tasks for Day 2, briefly explain the Learning Targets:

- Identify independent and dependent variables in a function
- Use words, tables, and graphs to represent functions
- Make sense of descriptions and graphs of functions, and explain what they tell about situations

Distribute graphing calculators and Student Task Statements:

- *Be Right Back!* (Unit 4, Lesson 1, Task 1.2)
- *Talk About a Function* (Unit 4, Lesson 1, Task 1.3)
- *The Backyard Pool* (Unit 1, Lesson 1, Task 1.4).

1. Ask students to individually read task 1.2 *Be Right Back!* Make sure the students understand what the task is asking them to do.
2. Assign each group of students one of the days to analyze. Ask students to collectively work on their graph and to answer the first question.
3. Circulate the room as the students work in their groups on the graph. Notice if any students create a graph with a vertical value greater than 5, or a graph that does not go through the two given points.
4. Engage students in a whole-class discussion by asking if it is possible to tell how many times the dog changed directions while walking around? Why or why not?

5. Discuss with the whole-class the features of their graphs. What is the input and what is the output? How do you know? Is it possible for the distance to be the input and the time to be the output? Would it be a function? Why or why not?
6. Introduce terms: independent variable and dependent variable. Relate the terms to the tasks, 1.1 *Bagel Shop* and 1.2 *Be Right Back!*
7. Arrange students in groups of 2 and tell the students they will be analyzing two pairs of quantities from a familiar situation in task 1.3: *Talk about a Function*. **Tell partners they must each choose a different pair of quantities to analyze**.
8. Read task 1.3 with the students and make sure all students understand the directions. Give the students time to decide which pair of quantities each will take and allow quiet time for the students to work on the task.
9. Circulate the room as the students work on the task. Look for students who draw different graphs for the same situation.
10. In pairs, have students share their functions and representations. Tell the partner who is listening to listen for the following information: How did the partner decide which quantity is independent (the input) and which is dependent (the output)? How did the partner decide what makes _____ a function of _____? How does the graph represent the relationship between _____ and _____? How was the scale for each axis on the graph determined?
11. Whole-class discussion: Select 2 students who drew different graphs for the same situation and ask them to share. How did the students decide which quantity was the input and which was the output? How did the students decide what the graph should look like?
12. Lesson Synthesis – Whole-class Discussion: Refer back to the last two tasks (1.2 and 1.3):
    - In the situation that involved time and number of barks, suppose the number of barks is the input and the time is the output. Is it possible that when the number of barks is 6, both 15 seconds and 16 seconds have passed? (YES)
    - Can the time be a function of the number of barks? Why or why not? (NO, because for any number of barks, more than one possible time could be the output.)
    - In the situation that involved time and the total distance walked by the dog, suppose the distance is the input and the time is the output. Is it possible that when the dog has walked 27 feet, 50 seconds, 55 seconds, and 60 seconds have passed? (YES)
    - Can the time be a function of the total distance walked? Why or why not? (No, because for any number of feet walked, many possible numbers of seconds could have passed.)
13. Student Journal – Check for Understanding: Direct students to read the 1.4 *The Backyard Pool* Student Task Statement and respond to the three questions in their journals.

Share, Discuss, and Analyze Phase

Essential Understanding #1:

A relationship between two quantities is a function if there is only one possible output for each input.

Share: On Day 1, students are given a situation where multiple individuals have calculated a different price for 13 bagels. After the students have projected how each individual could have a different price, the students share and justify within their group and the whole-class their thinking.

Discuss: After completing the table giving the prices of 1 to 13 bagels for the shopkeeper and the customers, the students discuss what the graphs of the data might look like. It is during this discussion that students develop their understanding of what makes a function.

Analyze: On Day 1, the students are asked to analyze sets of data to determine which set is a function. They are also given a new situation to analyze at the closure of the lesson to make sure the concept of a function is truly learned.

Essential Understanding #2:
Words, tables, and graphs can represent relationships that are functions, including identifying the independent and dependent variables.
Share: On Day 2, students analyze two pairs of quantities from a familiar situation. After working independently, the students share their functions and representations.
Discuss: In order to bring correct mathematical language forward, the students engage in a whole-class discussion for Lesson Synthesis
Analyze: At the closure of the lesson on Day 2, the students are given a new situation, *The Backyard Pool*, to analyze and demonstrate their understanding of what makes a function.

Task In Action

The video clips below provide a demonstration of the task being implemented in a classroom as it aligns with the Effective Mathematics Teaching Practice indicated. These clips should be used by the teacher to model the implementation of the task in his or her classroom.

- Implement Tasks That Promote Reasoning and Problem Solving:
  - **Video Clip #1**
- Use and Connect Mathematical Representations:
  - **Video Clip #2**
- Support Productive Struggle in Learning Mathematics:
  - **Video Clip #3**