WEST VIRGINIA
DEPARTMENT OF EDUCATION

MATHEMATICS

GRADE 9

Interpreting and Creating Graphs: Flag Raising

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Task Title: Functions: *Interpreting and Creating Graphs - Flag Raising*
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Quarter: 2

Rationale for Lesson and Associated Tasks

In this Illustrative Mathematics lesson, students interpret and create graphs of functions that are less well defined but model complex real-life situations. The lesson includes a warm-up activity to review properties of graphs; two tasks in which students interpret and sketch graphs based on video clips of the motion of flag raisings; and two tasks in which students explore the rate of change of a function in terms of two different real-life situations.

Graphs are widely used for learning about many topics, such as weather, history, economics, psychology, physics, genetics, and astronomy. Yet national and international assessments show that although most students can identify values on a graph (e.g., the temperature at noon was 59 degrees), many have trouble identifying what trends are shown on the graph, the key points where some phenomenon changed dramatically, the rate of change indicated by the points, and other less obvious information. (*Teaching Graph Literacy Across the Curriculum*, National Science Teachers Association [NSTA] Science Scope, Vol. 38, No. 6, February 2015)

It is important for students to learn about interpreting graphs so they can understand creating graphs which is a fundamental part of Algebra. In 2000, the National Council of Teachers of Mathematics identified the development of skills for interpreting and creating graphs as one of the most important objectives of mathematics education. Graphs are powerful because they allow students to visualize mathematical concepts. From this viewpoint, graphs are similar to other representations such as pictures, images, and diagrams. However, graphs are even more effective because they can also provide students with visual representations of mathematical relations, functions, and their properties. (*Examining How Teachers Use Graphs to Teach Mathematics*, Journal of Education and Training Studies Vol. 3, No. 2; March 2015)

Lesson and Associated Tasks Overview

Illustrative Mathematics: Algebra I – Unit 4 Functions – Lesson 8 *Interpreting and Creating Graphs*
https://curriculum.illustrativemathematics.org/HS/teachers/1/4/8/preparation.html

Review all components (Teacher Preparation, Teacher Lesson Guide, Practice, and Student Lesson) of the task thoroughly. Both digital and print formats are available for most tasks.

By grade 9, students have had multiple opportunities to interpret graphs of functions and to create them (primarily by plotting known input-output pairs of a function or by using descriptions of the situation). Students have also acquired essential vocabulary to communicate about graphs of functions and use average rate of change to measure how a function changes.

In this lesson, students apply these insights and skills to interpret or create graphs of functions that are less well defined and that model real-life situations that are more complex. Information about the functions is presented in the form of verbal descriptions, video clips, and images. More ambiguity is involved here than in cases students have previously encountered, so they will need to persevere in sense-making and problem-solving. At times, the information given may be inadequate, so students will

need to make assumptions and decisions in order to produce graphs that show the desired behaviors or meet certain requirements. Along the way, students engage in important aspects of mathematical modeling.

This lesson and associated tasks are scheduled to be completed over two class periods per the suggested sequence. The following is an overview of the lessons:

Day 1

1. Introduce the Learning Targets for the lesson
2. Distribute Lesson 8 Interpreting and Creating Graphs task handouts: 8.1 *Which One Doesn't Belong: Temperature Over Time,* 8.2 *Flag Raising (Part 1),* and 8.3 *Flag Raising (Part 2)*
3. Introduce, play, and discuss YouTube video *What Are the Different Parts of a Graph?* (6:21) (click here)
4. Engage students in the warm-up activity, 8.1 *Which One Doesn't Belong: Temperature Over Time*
5. Show pictures and describe the process of raising a flag or show *Flag Raising Ceremony* (0:44) video clip: (click here)
6. Explore interpreting graphs with students in task 8.2 *Flag Raising (Part I)*
7. Show flag raising videos at both half and full video speeds
   - *Raising a Flag (Full Speed)* (click here)
   - *Raising a Flag (Half Speed)* (click here)
8. Engage students in task 8.3 Flag Raising (Part 2) replaying the videos as needed
9. Display all student graphs and ask students to participate in a Gallery Walk. Ask students to create a list or graphic organizer of how the graphs are alike and how they are different
10. Student journal: Answer question #2 of task 8.3 - What does the average rate of change tell us about the flag?

Day 2

1. Review the Learning Targets and tasks from Day 1
2. Introduce the Learning Targets for Day 2 tasks
3. Distribute Lesson 8 Interpreting and Creating Graphs handouts: 8.4 *Two Pools* and 8.5 *The Bouncing Ball*
4. Arrange students in groups of 2 and explore creating graphs of functions based on verbal descriptions in task 8.4 *Two Pools*
5. Engage students in task 8.5 *The Bouncing Ball*
6. Watch YouTube video *Interpreting Graphs* (12:41) (click here)
7. Student Journal Prompt: A child tosses a baseball up into the air. On its way down, the ball gets caught in a tree for several seconds before falling to the ground. Sketch a graph that represents the height of the ball, h, as a function of time, t. Explain why the graph demonstrates the motion of the ball.

West Virginia College- and Career-Readiness State Standards (Integrated)

M.1HS.15

For a function that models a relationship between two quantities, **interpret key features of graphs and tables in terms of the quantities** and **sketch graphs showing key features given a verbal description of the relationship.** Key features include: **intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums;** symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.

M.1HS.17
Calculate and **interpret the average rate of change of a function** (presented symbolically or as a table) over a specified interval. **Estimate the rate of change from a graph.** Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.

West Virginia College- and Career-Readiness State Standards (Traditional)

M.A1HS.21
For a function that models a relationship between two quantities, **interpret key features of graphs and tables in terms of the quantities**, and **sketch graphs showing key features given a verbal description of the relationship**. Key features include: **intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums**; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.

M.A1HS.23
Calculate and **interpret the average rate of change of a function** (presented symbolically or as a table) over a specified interval. **Estimate the rate of change from a graph**. Instructional Note: Focus on linear functions and exponential functions whose domain is a subset of the integers. The Unit on Quadratic Functions and Modeling in this course and the Algebra II course address other types of functions.

Mathematical Habits of Mind (MHM)

The tasks relate to most of the Standards for Mathematical Practice, but have an emphasis on:

MHM. 1. Make sense of problems and persevere in solving them.
MHM. 2. Reason abstractly and quantitatively.
MHM. 3. Construct viable arguments and critique the reasoning of others.
MHM. 4. Model with mathematics

Mathematics Teaching Practices to Support Student Growth

The tasks relate to most of the Mathematics Teaching Practices, but have an emphasis on:

MTP.2. Implement tasks that promote reasoning and problem solving.
MTP.3. Use and connect mathematical representations.

Essential Understandings

- Functions are a mathematical way to describe relationships between two quantities that vary.
- Representing functions as graphs is an alternative way of depicting and analyzing patterns of change.
- Real-world scenarios can be modeled mathematically (graphed) in order to analyze, interpret, and evaluate data.

Set–up Phase

1. Become an Expert Regarding All Lessons and Associated Tasks Content
It is imperative that the teacher becomes very familiar with all *Interpreting and Creating Graphs* (Illustrative Mathematics - Algebra I - Unit 4 - Lesson 8) materials: Teacher Preparation, Teacher Lesson Guide, Practice, Student Lesson, Vimeo videos, digital app, and student handouts. Lesson 8 has several associated tasks and some tasks may be delivered via printed or digital resources.

The implementation of the tasks may vary class to class depending on the time frame available for the

lesson, and the levels of student engagement and understanding. Suggestions for *Students with Disabilities* and *English Language Learners* are provided within the lesson.

2. Establish Small Groups

Interpreting and Creating Graphs tasks and activities engage students in both individual and group thinking. Working individually, in pairs and in small groups, students will explore and share their reasoning about graphing real-world scenarios. Small group instruction has significant impact on student achievement (Hattie, 2009) and allows teachers to work more closely with each student. This type of instruction provides the opportunity to monitor students' learning and identify gaps in the development of students' math skills.

Teachers will need to instruct students on how to work in small groups. The first step in the process is to establish ground rules and norms for interaction. Students must have a part in making the rules and these guidelines must be enforced by both teachers and students. Ground rules should encourage positive collaborative behaviors among all students. Guidelines/ground rules need to be posted in the classroom so students can readily refer to them. If students or teachers believe that additional rules are needed, they can be added later.

Teachers should assign groups intentionally based on skills and/or backgrounds. Skill levels, leadership skills, and personalities must all be considered when creating small groups. Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis. Observations of leadership skills; personalities; ability to take criticism; ability to question; and the ability to think deeply about a task or problem will prove to be extremely helpful when creating small groups for this unit. This strategy minimizes the chance that high ability students will flock together leaving others out, allows teachers to create more diverse groups, and creates opportunities for students to work with peers with whom they otherwise might not have interacted.

Small group instruction gives teachers an opportunity to assess more closely what each student can do and build strategic plans around those assessments. Students who struggle to ask questions and participate in a whole group setting may thrive in a small group where they feel more comfortable and less overwhelmed. Furthermore, small group instruction tends to proceed at a fast pace, which typically helps students maintain focus and enjoy the learning experience.
Source: *Small Group Instruction: How to Make it Effective*, CORE: Excellence in Education Blog, September 27, 2018.

3. Develop Open-Ended Questions

Teachers of *Flag Raising*: *Interpreting and Creating Graphs* should use open-ended questions to support and scaffold the lesson and associated tasks for their students. These questions should purposefully direct students towards provided information, previously learned content, and similarities and differences in their work versus other group members. Within the context of open-ended mathematical tasks, teachers should select questions with four purposes in mind:

A. *Starter questions*

Starter questions focus students' thinking in a general direction and give a starting point. Starter questions might include:

- What units of measurement could make an unrealistic graph more realistic? (Task 8.2)
- What scale could you use for the axes of your graph? (Task 8.3)

B. *Questions to stimulate mathematical thinking*

These questions focus on particular strategies aiding the formation of a strong conceptual network.

The questions can serve as a prompt when students become 'stuck'. Questions to stimulate mathematical thinking might include:

- What differences might make a graph not belong? (Task 8.1)
- What quantities might you need to sketch the graph? (Task 8.2)
- How are the graphs alike? (Task 8.2)
- How are the graphs different? (Task 8.2)
- When did constant intervals occur in the graphs? (Task 8.3)
- What does the constant interval tell you about the motion of the flag? (Task 8.3)

C. *Assessment questions*

Assessment questions ask students to explain what they are doing or how they arrived at a solution. They allow the teacher to see how students are thinking, what students understand, and at what level students are operating mathematically. These open-ended questions are best asked after time has been given to make progress with the problem, to record some findings, and perhaps to have achieved at least one solution. Assessment questions might include:

- How do you know that each graph represents a function? (Task 8.2)
- Why does the vertical line graph not represent a function? (Task 8.2)
- What did you discover that affected the height of the water in the large and small pools? (Task 8.4)
- What happens to the output value, *h*, when the input value, *t,* increases? (Task 8.5)

D. *Final discussion questions*

These questions draw together the efforts of the class and prompt the sharing and comparison of strategies and solutions. This is a vital phase in the mathematical thinking processes. It provides further opportunity for reflection and realization of mathematical ideas and relationships. It encourages students to evaluate their work. Final discussion questions might include:

- Is everyone's graph the same? Why or why not? (Task 8.3)
- How would the graph of the large pool change if one hose was moved from the small pool to the large pool? How do you know? (Task 8.4)
- What would happen to the graph of the small pool if one hose was moved from the small pool to the large pool? How do you know? (Task 8.4)

*Additional questions are provided in the Teacher Guide.

Source: *Using Questioning to Stimulate Mathematical Thinking*, NRICH, February 2011.

4. Gather Materials

- Computer and presentation device
- Internet access
- Pictures (digital or paper) of a flag raising or *Flag Raising Ceremony* video clip (click here)
- Videos (Online):
  - *Raising a Flag (Full Speed)* (click here)
  - *Raising a Flag (Half Speed)* (click here)
  - *Interpreting Graphs* (click here)
- Illustrative Mathematics Student handouts:
  - *Which One Doesn't Belong: Temperature Over Time* (Algebra I - Unit 4, Task 8.1)
  - *Flag Raising – Part I* (Algebra I – Unit 4 Task 8.2)
  - *Flag Raising – Part 2* (Algebra I – Unit 4 Task 8.3)
  - *Two Pools* (Algebra I – Unit 4 Task 8.4)
  - *The Bouncing Ball* (Algebra I – Unit 4 Task 8.5)

5. Anticipated Common Student Misconceptions

Day 1 Task 8.3 *Flag Raising – Part 2*

Some students may have trouble starting their graphs because they do not know what upper limits to use for the axes. Ask them to watch the videos again and try to gather information that may help them decide on the upper limits. Assure them that some estimation and decision making are necessary.

Day 2 Task 8.4 *Two Pools*

Some students might mistakenly think that when the pools are "full," the water in each pool has reached the same height. Remind students that the two pools have different heights, so it takes different heights of water to make them full.

Day 2 Task 8.5 *The Bouncing Ball*

Some students may mistake the horizontal axis on the graph to represent horizontal distance rather than time. Because the movement of the bouncing ball is primarily up and down (except toward the end, when it begins rolling), these students might sketch a graph that is essentially a series of overlapping vertical segments. Ask them to revisit the input variable and what the horizontal axis represents. Then, ask them to plot some points for different values of *t*.

Explore Phase

While the Grade 8 West Virginia College- and Career-Readiness Standards for Mathematics call for students to work informally with functions, students in grade 9 begin to refine their understanding and use of the formal mathematical language of functions. In grade 9, students come to understand the concept of a function, how to interpret characteristics of a function within a context, and how to represent functions in different ways.

In this lesson, students expand their understanding of functions, building on what they learned in grade 8. Students develop their capacity to represent, interpret, and use functions to make sense of quantities in situations and to solve problems. They are introduced to new tools for communicating about functions: average rates of change and mathematical terms for describing key features of graphs. They also develop their ability to gather information about a function from its graph, by connecting features of the graph to features of the situation and other representations, and to sketch a graph that tells the story about the function.

Grade 9 students will need ample opportunities (e.g. "bell ringers") to revisit the following Grade 8 standards prior to the introduction of the Interpreting and Creating Graphs lesson and tasks:

- **M.8.12**
  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
- **M.8.14**
  Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- **M.8.15**
  Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Prior Instruction/Knowledge

The study of proportional relationships in grade seven is a foundation for the study of functions that is introduced in grade eight and continues through higher mathematics. In grade seven, students learn to determine if two quantities represent a proportional relationship. Proportional reasoning is a transitional topic that falls between arithmetic and algebra. Underlying the progression from proportional reasoning through algebra and beyond is the idea of a *function*—a rule that assigns to each input exactly one output.

The concept of functions is a critical area of instruction in grade eight. Students are introduced to functions and learn that proportional relationships are part of a broader group of linear functions. The same kinds of tables and graphs that students use in grade seven to recognize and represent proportional relationships between quantities are used in grade eight when students compare the properties of two functions that are represented in different ways (e.g., numerically in tables or visually in graphs). Students also compare the properties of two functions that are represented algebraically or verbally.

Please review the following:

Educators Guide for Mathematics: Grade 7 (pages 6-7) (click here)
Educators Guide for Mathematics: Grade 8 (pages19-23) (click here)

Prerequisite Skills

- Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form.
- Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions.

Supporting Skills

- Determine the ratio or rate of change of a relation given a table or graph.
- Use and interpret function notation in number and word problems; determine a value of the function given an element of the domain.

Impending Skills

- Graph quadratic functions. Identify and interpret the intercepts, maximum, minimum, and the axis of symmetry.
- Graph absolute value functions and their corresponding inequalities.
- Write the equation of and graph exponential equations or functions, including f(x) = ab^x and f(x) = a(1+r)^x, in number and word problems; identify and interpret critical values.
- Write and graph special functions (step, constant, and piecewise) and identify the domain and range.
- Describe the intervals for which a function is increasing or decreasing.

Source: *The Quantile Framework for Mathematics* 2020 MetaMetrics Inc.
https://metametricsinc.com/educators/quantile-for-educators/

Implementation Phase

Day 1

1. Introduce Learning Targets for the lesson
2. Distribute Lesson 8 Interpreting and Creating Graphs task handouts: 8.1 *Which One Doesn't Belong: Temperature Over Time,* 8.2 *Flag Raising (Part 1),* and 8.3 *Flag Raising (Part 2)*

3. Introduce, play, and discuss YouTube video *What Are the Different Parts of a Graph?* (6:21) (click here)
4. Engage students in the warm-up activity, 8.1 *Which One Doesn't Belong: Temperature Over Time*
5. Show pictures and describe the process of raising a flag or show *Flag Raising Ceremony* (0:44) (click here)
6. Explore interpreting graphs with students in task 8.2 *Flag Raising (Part I)*
7. Show flag raising videos at both half and full video speeds
   - *Raising a Flag (Full Speed)* (click here)
   - *Raising a Flag (Half Speed)* (click here)
8. Engage students in task 8.3 *Flag Raising (Part 2)* replaying the videos as needed
9. Display all student graphs and ask students to participate in a Gallery Walk. Ask students to create a list or graphic organizer of how the graphs are alike and how they are different
10. Student journal: Answer question #2 of task 8.3 What does the average rate of change tell us about the flag?

Day 1 Teacher Notes:

To introduce the day's tasks, briefly explain the Learning Targets:

- Analyze and compare the properties of graphs.
- Make sense of important features of a graph and explain what they mean in a situation.
- Explain the average rate of change of a function in terms of a situation.

Distribute the student handouts:

- 8.1 *Which One Doesn't Belong: Temperature Over Time*
- 8.2 *Flag Raising (Part I)*
- 8.3 *Flag Raising (Part 2)*

1. Arrange students in groups of 2-4 members.
2. Introduce the YouTube video *What Are the Different Parts of a Graph?* by asking students (whole-class) what they remember about the parts of a graph from grade 8. Record all student responses and display for all students to see. Be sure to record student language used (e.g. *x*-axis, *y*-axis, dependent variable, independent variable, origin).
3. Play the video.
4. Discuss in a whole-class setting what students now remember about the parts of a graph from grade 8 after watching the video. Record the responses. Compare with the previous list. Was there new information in the video? Did students correctly remember the parts of a graph from grade 8?
5. Ask students to look at the four graphs on **Task 8.1 *Which One Doesn't Belong: Temperature Over Time*.** Give students at least 2 minutes to study the graphs independently.
6. Ask students to share within their group which graph(s) do not belong and the reasoning for why the selected graph(s) do not belong.
7. Engage students in a whole-class discussion. Ask each group to share one reason why a particular graph does not belong. Record and display all student responses. After each response, ask the class if they agree or disagree. Be sure to listen for correct reasoning as there are no single correct answers. Listen for and encourage accurate use of mathematical language as presented in the video, *What Are the Parts of a Graph?*
8. Present students with pictures of a flag raising or play the YouTube video, *Flag Raising Ceremony*. Discuss with students the procedures for correctly raising a flag.
9. Ask students to look at the graphs on task **8.2 Flag Raising (Part 1)** and to independently think about:

- What might be reasonable units to use for the axes of the graphs?
- How do we know if each graph represents a function?

Have students share their thinking in their groups. Follow up with a whole-class discussion.

10. Ask student groups to read and complete questions #1a and #1b. Each group member should take a turn interpreting a graph in the first question. Circulate the room providing students with assistance as needed.
11. Engage students in a whole-class discussion about:
    - Which graph(s) appear to be most realistic and why?
    - Which graph(s) appear to be the least realistic and why?
12. Ask students to independently read and complete questions #2a and #2b. Circulate the room providing students with assistance as needed.
13. Have students share their thinking about #2a in their groups. Follow up with a whole class discussion of #2b: Does the graph represent a function or not and why?
14 Introduce students to task **8.3 *Flag Raising (Part 2)***. Tell students to watch the videos of a flag being raised and to sketch a possible graph to represent the height of the flag as a function of time (Question #1). Explain that their graphs do not need to be precise and that estimations are required. The graph needs to reasonably capture the movement of the flag in the videos.
    - *Raising a Flag (Full Speed)* (click here)
    - *Raising a Flag (Half Speed)* (click here)
15. Replay the flag raising videos as many times as needed until all students' graphs are completed. Display all student graphs.
16. Ask students to complete a Gallery Walk and create a list or graphic organizer of similarities and differences among the graphs displayed.
17. Ask students to complete Question #2 in their Student Journals: *Explain what the average rate of change tells us about the flag.* Students will need their graphs to estimate the average rate of change from the time the flag starts moving to the time it stops before completing their journal entry.

Day 2

1. Review the Learning Targets and tasks from Day 1
2. Introduce the Learning Targets for Day 2 tasks
3. Distribute Lesson 8 Interpreting and Creating Graphs handouts: 8.4 *Two Pools* and 8.5 *The Bouncing Ball*.
4. Arrange students in groups of 2 and explore creating graphs of functions based on verbal descriptions in task 8.4 *Two Pools*
5. Engage students in task 8.5 *The Bouncing Ball*
6. Watch YouTube video *Interpreting Graphs* (12:41) (click here)
7. Student Journal Prompt: A child tosses a baseball up into the air. On its way down, the ball gets caught in a tree for several seconds before falling to the ground. Sketch a graph that represents the height of the ball, *h*, as a function of time, *t*. Explain why the graph demonstrates the motion of the ball.

Day 2 Teacher Notes:

Organize students into pairs. Review the Day 1 Learning Targets and tasks by having students share their journal entries with their partners.

To introduce the tasks for Day 2, briefly explain the Learning Targets:

- Given a description or a visual representation of a situation, sketch a graph that shows

important features of the situation.

- Explain the average rate of change of a function in terms of a situation.

Distribute the student handouts:

- 8.*4 Two Pools*
- 8.5 *The Bouncing Balls*

1. Ask students to read the task **8.4** ***Two Pools*** and clarify any questions students may have about the task.
2. Tell students to independently sketch the graphs for the first situation. Circulate the room as students work and provide assistance as needed.
3. Ask students to share their graphs with their partner – discussing their assumptions about the situation and the reasonableness of the graph based on their assumptions. Give students the opportunity to revise their graphs based on the discussion with their partners.
4. Lead a whole-class discussion by asking students to share their graphs. Extend the discussion by asking what assumptions students made about the situation:
   - The water in both hoses flows at a constant rate. Did you assume the constant rate in one hose to be the same as the constant rate in the other hose?
   - Did you assume that the water for the two hoses was turned on at the same time?
   - What assumptions did you make about the shape and size of each pool? Did you assume the pools to have the same area for their base and just have different heights, or that they have different areas?
   - What assumptions did you make about how water was rising in each pool?
5. Ask students to independently sketch the graphs for Situations 2 and 3. Circulate the room as students work and provide assistance as needed.
6. Ask students to share their graphs with their partner – discussing their assumptions about the situation and the reasonableness of the graph based on their assumptions. Give students the opportunity to revise their graphs based on the discussion with their partners.
7. Engage students in a whole-class discussion of their graphs for Situations 2 and 3. Select students to share their graphs, assumptions, and the decisions made as they graphed Situations 2 and 3. Be sure to select graphs that correctly represent the same situation but look different due to the variation in the assumptions made.
8. Extend the discussion by asking:
   Q. How would the vertical values of the two graphs compare when the pools are full?
   A. The vertical value of the large pool would be up to 12 inches greater than that of the small pool because the large pool is 12 inches taller than the small pool.
   Q. When each pool is being filled by one hose and at the same rate, should the two graphs have the same slope? Why or why not? If not, which graph has a greater slope?
   A. No. The water in the smaller pool rises more quickly because the area of the pool is smaller, so the slope for that graph should be greater.
   Q. How would the graph of the large pool change when one hose was moved from the small pool to the large pool? Would its slope increase, decrease, or stay the same?
   A. The slope of the graph for the large pool would double because water is now rising at twice its previous rate. The graph for the small pool would stay constant, as the water is no longer increasing in height.

9. Introduce task **8.5 *The Bouncing Ball*** by explaining the video *Tennis Ball Drop* will show the motion of a ball dropped but played back at different speeds. Inform students they will be asked to graph the motion of the dropped tennis ball.
10. Ask students to think independently about what information from the video they will need to create the graph. Have students share their ideas with a partner and to create a list of information or quantities they will need from the video to create the graph.
11. In a whole-class discussion, ask the student pairs to share their lists. Ask students why they would need to know the information or quantities they have listed. Record and display all student responses.
12. Direct students to read task 8.5 *The Bouncing Ball* and to complete the table in Question 1. Clarify any student questions before students begin the task.
13. Ask students to share their table with a partner. Play and replay all the videos several times to help students independently sketch (as accurately as possible) a graph of the motion of the tennis ball in the coordinate grid of Question 1. An alternative approach would be to give students individual access to the Illustrative Mathematics interactive applet.
    - Tennis Ball Drop (Full Speed) (click here)
    - Tennis Ball Drop (Half Speed) (click here)
    - Tennis Ball Drop (Quarter Speed) (click here)

    Circulate around the room providing students with assistance as needed.
14. Have students share their graphs with a partner. Tell students to be prepared to explain any differences they observe between the graphs.
15. Select a few student teams to share their graphs with the whole class and to explain any differences they observed.
16. Discuss with students the terms: **vertical intercept, horizontal intercept, maximum value of a function** and **minimum value of a function**.
17. Direct students to read and complete task 8.5 Questions 2 and 3 independently. Circulate around the room providing students with assistance as needed.
18. Ask students to share their responses to Questions 2 and 3 with a partner.
19. Engage students in whole-class discussion:
    Q. If you only see the still images of the ball and not the video of the ball bouncing, can you accurately graph the height as a function of time? Explain your reasoning.
    A. No. Each still image gives the position of the ball at a different time, or one point on the graph that represents height as a function of time. Without the video, we would not know what is happening at the time the image is taken. Is the ball going up? Is the ball coming down? Is it at one of the peaks?
    Q. Which parts of the ball's movement did you find easier to plot more precisely? Which parts required you to estimate?
    A. The original position of the ball before it is released and the points when the ball hits the ground are easier to plot because they are more obvious. The points when the ball reached the peak and changed direction requires estimation. The points between the peaks and the bounces are also estimated.
    Q. In your graph, the horizontal intercepts are also the minimums of the graph. Why?
    A. The ball could not have a height that is less than 0. The lowest point it could be is 0 feet.
    Q. What happened to the output value, h, as the input value, t increased?
    A. Overall, the output value decreased as t increased, but there are intervals – when the ball bounces up – in which the height of the ball increases as time increases.
20. Use the YouTube video *Interpreting Graphs* (12:41) as a summarizing activity for the lesson. Pause the video before the narrator works through the questions and ask students to complete the

question independently or in pairs. Restart the video once all students have completed their work. Pause the video at the end of the narrator's explanation. Engage students in a whole-class discussion about the problem and clarify any student misunderstandings before moving to the next question.

Interpreting Graphs (12:41) (click here)

21. Student Journal prompt: A child tosses a baseball up into the air. On its way down, the ball gets caught in a tree for several seconds before falling to the ground. Sketch a graph that represents the height of the ball, *h*, as a function of time, *t*. Explain why the graph demonstrates the motion of the ball.

Share, Discuss, and Analyze Phase

Essential Understanding #1*:*

Functions are a mathematical way to describe relationships between two quantities that vary.

Share – In **Task 8.2 *Flag Raising* (*Part 1)***, students explore the effects of time on the height of a flag through six different graphs. Working in small groups, each student must interpret at least one of the graphs in the context of raising a flag and make a case to the members of their group for whether the graph is realistic.

Discuss – At the end of the **Task 8.2 *Flag Raising* (Part 1)** exploration, whole-class discussion occurs where students explain and justify which graph(s) are most realistic and which graph(s) are least realistic. Students are then given another graph that relates time and height, but the graph is a vertical line to the *x*-axis. The class discusses whether the graph can represent the time and height of the flag (is it realistic) and if it represents a function.

Analyze – After reviewing the different parts of a graph, students in task **8.1 *Which One Doesn't Belong: Temperature Over Time*,** analyze four different graphs to determine why each does not belong. As there is no single correct answer for each graph, be attentive to students' explanations, and ensure the reasons given are valid.

Essential Understanding #2:

Representing functions as graphs is an alternative way of depicting and analyzing patterns of change.

Share – In task **8.4 *Two Pools,*** students create three different graphs of functions based on verbal descriptions of three different situations involving the filling of a large and a small backyard pool. Although students create their initial graphs independently, they also share their graphs with a partner – discussing their assumptions and the reasonableness of the graph based on those assumptions. After sharing and receiving the input of a partner, students are given time to revise their graphs.

Discuss – In task **8.4 *Two Pools***, students participate in paired and whole-class discussions concerning the three graphs of functions they create based on verbal descriptions of a situation. The modeling demand is significant as students receive little quantitative information and therefore need to make more assumptions and decisions. Extended whole-class discussions occur by asking students about the assumptions made:

- Q. The water in both hoses flows at a constant rate. Did you assume the constant rate in one hose to be the same as the constant rate in the other hose?
- Q. Did you assume that the water for the two hoses was turned on at the same time?
- Q. What assumptions did you make about the shape and size of each pool? Did you assume the pools to have the same area for their base and just have different heights, or that they have different areas?
- Q. What assumptions did you make about how water was rising in each pool?

Answers will vary; be attentive to students' explanations and ensure their reasoning is valid.

Analyze – In task **8.4 *Two Pools*,** students are engaged in an analysis of patterns of change displayed in their graphs:

Q. How would the vertical values of the two graphs compare when the pools are full?

A. The vertical value of the large pool would be up to 12 inches greater than that of the small pool because the large pool is 12 inches taller than the small pool.

Q. When each pool is being filled by one hose and at the same rate, should the two graphs have the same slope? Why or why not? If not, which graph has a greater slope?

A. No. The water in the smaller pool rises more quickly because the area of the pool is smaller, so the slope for that graph should be greater.

Q. How would the graph of the large pool change when one hose was moved from the small pool to the large pool? Would its slope increase, decrease, or stay the same?

A. The slope of the graph for the large pool would double because water is now rising at twice its previous rate. The graph for the small pool would stay constant, as the water is no longer increasing in height.

Essential Understanding #3:

Real-world scenarios can be modeled mathematically (graphed) in order to analyze, interpret, and evaluate data.

Share – In task **8.5 *The Bouncing Ball,*** students watch recordings of the motion of a dropped tennis ball at full, half, and quarter speed. Before watching the videos, students share their ideas with a partner and create a list of information or quantities they will need from the video to create a graph depicting the motion of the tennis ball.

Discuss – In paired and whole-class discussions of task 8.5 *The Bouncing Ball*, students share their lists of information or quantities needed from the video to graph the motion of a dropped tennis ball. Students discuss why they would need to know the information or quantities they have listed.

Analyze – Students analyze how the movement of the dropped ball is translated into a graph and the connections between the graph and situation:

Q. Which parts of the ball's movement did you find easier to plot more precisely? Which parts required you to estimate?

A. The original position of the ball before it is released and the points when the ball hits the ground are easier to plot because they are more obvious. The points when the ball reached the peak and changed direction requires estimation. The points between the peaks and the bounces are also estimated.

Q. In your graph, the horizontal intercepts are also the minimums of the graph. Why?

A. The ball could not have a height that is less than 0. The lowest point it could be is 0 feet.

Q. What happened to the output value, h, as the input value, t increased?

A. Overall, the output value decreased as t increased, but there are intervals – when the ball bounces up – in which the height of the ball increases as time increased.

Task in Action

The video clips below provide a demonstration of the task being implemented in a classroom as it aligns with the Effective Mathematics Teaching Practice indicated. These clips should be **used by the teacher as a model for the implementation** of the task in his or her classroom. The video clips are **NOT** for student viewing.

- Implement Tasks That Promote Reasoning and Problem Solving:
  - Video Clip
- Use and Connect Mathematical Representations:
  - Video Clip