WEST VIRGINIA
DEPARTMENT OF EDUCATION

MATHEMATICS

GRADE 8

The Two Storage Tanks

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Task Title: The Two Storage Tanks Task
Grade or Content Area: 8th Grade
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Original Task Creator: NAEP Released Item, 2003
Quarter: 2

Rationale for Lesson and Associated Tasks

This task gives students the opportunity to extend their knowledge of linear equations and linear functions. They begin exploring the concept of a system of two linear equations, learning that the solution to the system is represented graphically by the point(s) of intersection of the lines, with the ordered pair(s) of the point(s) making both equations true. Students use multiple representations (graphs, contexts, tables, and equations) to demonstrate an understanding of a real-world situation and proceed along a path that will lead them to a solution.

Lesson and Associated Tasks Overview

Use the following link and path to access the NAEP Released Item. The link provides the teacher with the ability to print the problem and to see sample solutions and notes. (click here)

- Search Questions (Note: It may be necessary to wait after selections are made; the website is not quick to respond.)
- Select Mathematics, Grade 8,
- Select Algebra, short constructed response, hard, Year 2003; Select OK at top of the page
- Select 2003-8M10 #13 (You might have to scroll through a few questions.)
- Then Show Question (It will appear at the bottom of the page.)

By linking to the problem, the teacher will be able to print the problem and to see sample solutions and notes.

Students are provided with a visual representation, a graph, which shows the changing water level in two storage tanks over a period of time. One tank is losing water at a steady rate and the other tank is gaining water. Students will determine the point in time when the two tanks have the same amount of water.

Day 1

- Give students a copy of the graphs and give them private think time to make observations.
- Discuss what students notice, reviewing key concepts about graphs and equations of lines.
- Present the problem. "When will the tanks hold the same amount of water?"
- Allow time for students to work on the problem with a partner.
- As partners finish the problem, give them extension problems.
- Analyze the work by discussing various pathways and extensions of the problem.

Day 2 – as needed to continue working on/analyzing the problem

West Virginia College- and Career-Readiness State Standard

M.8.10
Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. (e.g., 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.)

c. Solve real-world and mathematical problems leading to two linear equations in two variables. (e.g., Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.)

M.8.14
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Mathematical Habits of Mind (MHM)

*While several MHM are listed, the one MHM in bold font is the focus of the lesson and associated tasks.

MHM1. Make sense of problems and persevere in solving them.
MHM4. Model with mathematics. *
MHM6. Attend to precision.

*In this lesson, students **model** a real-world problem situation using different representations, that is, contextually, graphically, symbolically (equations), and in a table. They begin to make decisions about which model best serves the purpose of helping them solve the problem.

Mathematics Teaching Practices to Support Student Growth

Implement tasks that promote reasoning and problem solving.
Use and connect mathematical representations.
Facilitate meaningful mathematical discourse.

Essential Understandings

- A linear function is defined by a rate of change that is constant. The function can be represented by a table, an equation, a graph, or in words.
- Systems of linear equations can be used to model and solve real-world problems.

Set-up Phase

1. It is essential that the teacher complete the task prior to classroom instruction. This provides an opportunity to explore the problem and to anticipate the challenges that students may encounter. In

working through the task, the teacher should anticipate various solution pathways and how those pathways might be connected so that various relationships can be exposed.
The teacher should become familiar with the material in the **Analyze** sections of this document to find the intention of this task.

2. Establish Partners and Table Teams
Skill levels, leadership skills, and personalities are all considered when creating small (e.g. partners, table teams of four students) groups. Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis. Through weeks of teacher observation and making note of leadership skills, personalities, the ability to take criticism, to question, and think deeply about a task or problem will prove to be extremely helpful when creating small groups for this lesson and associated tasks.

When creating the groups, all these factors help to eliminate the potential situation in which one student takes the lead and makes the decisions for the group. In this scenario, one student is gaining all the benefits of the task, while the others do not. If a student is not engaged in conversation, this lesson and associated tasks will not be beneficial in helping all students to have meaningful discussions about the mathematics involved nor in analyzing the relationships inherent to the tasks. Students will be working together when placed as partners to complete this task. The goal is for all students to participate in discussion and decision-making.

3. Develop Open Ended Questions
Teachers should create a list of open-ended questions designed to support and scaffold the task for struggling students. These questions should purposefully direct students towards necessary decisions and/or provided information, and lead students to previously learned content. Some questions might include:

- What is the rate at which each tank is increasing or decreasing? How can you use this information to help you?
- How do you know that the lines can be extended?
- How can you be certain that your lines are exactly where they should be?
- What do you need to know in order to write an equation for the lines? Where and how can you find this information?
- How will you represent a decrease in the equation?
- Is there another way you might find the solution without writing equations?
- How can you check to see if your solution is correct?

4. Gather Materials

- Copies of “The Two Storage Tanks Task”
- Straightedges/rulers (available if requested)
- Blank paper (optional)
- Graph paper (optional)
- Online graphing programs (Desmos, for instance) can be used to check and enhance this work (Optional)
- Graphing calculators (optional) can be used

5. Anticipated Common Student Misconceptions

Teachers should create a list of possible student misconceptions that may occur. Anticipated misconceptions/errors:

- If students use the graph to find the point of intersection, they may not extend the line segments accurately.
- Students may try to calculate the amount water flowing from one tank into the other.
- Student may not take in account the scale of the y-axis scale, leading them to an incorrect rate of change.
- Students might mix up the structure of the equations – e.g., y = bx + m (rather than y = mx + b or y = b + mx)
- Students may confuse the independent and dependent variable when setting up a table.
- Students may confuse the y-intercept (b) and the slope (m) when writing the equation.
- Students may try to write one equation only, instead of one for each line.
- Students may ignore the fact that one of the lines has a negative slope.

6. Plan for Early Finishers

Teachers should also plan lesson extensions for students who quickly complete the task.

- How much water is each tank when they have the same amount?
- What is another representation you can use to find the solution?
- When will Tank T be empty?
- How much water is in Tank T at 7 hours?
- How much water is in Tank T at 7 hours? At 20 hours? What are various ways you can determine this?
- Write the equations using a different form.
- Is it true that there will be 1000 gallons of water in Tank W at 16 hours?
- Is it true that there will be 925 gallons of water in the tank at 25 hours?
- How can you tell by looking at the equations that the lines are going to intersect?
- How might you find the solution by only using the equations you wrote?

Prior Instruction/Knowledge:

Please review the following:

Educators Guide for Mathematics: Grade 8 (pages 6 – 23, pdf pages 8-25) (click here)

Prerequisite Skills

- Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions.
- Generate a set of ordered pairs using a rule that is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form.
- Solve linear equations using the associative, commutative, distributive, and equality properties and justify the steps used.
- Locate points in all quadrants of the coordinate plane using ordered pairs in number and word problems.
- Determine the ratio or rate of change of a relation given a table or graph.
- Use a coordinate grid to solve number and word problems. Describe the path between given points on the plane.

Supporting Skills

- Write the equation of and graph linear relationships given the slope and y-intercept.
- Write the equation of and graph linear relationships given two points on the line.
- Identify and interpret the intercepts of a linear relation in number and word problems.
- Describe the slope of a line given in the context of a problem situation; compare rates of change in linear relationships represented in different ways.

Impending Skills

- Solve systems of linear inequalities.
- Write and solve systems of linear equations in two or more variables algebraically in number and word problems.
- Determine whether a system of equations has one solution, multiple solutions, infinitely many solutions, or no solution using graphs, tables, and algebraic methods; compare solutions of systems of equations.
- Determine the effects of changes in slope and/or intercepts on graphs and equations of lines.
- Interpret and compare properties of linear functions, graphs, and equations.
- Convert between different representations of relations and functions using tables, the coordinate plane, and algebraic or verbal statements.
- Use matrices to solve systems of equations with 2 or more variables.
- Graph quadratic functions. Identify and interpret the intercepts, maximum, minimum, and the axis of symmetry.

Source: *The Quantile Framework for Mathematics*
https://metametricsinc.com/educators/quantile-for-educators/
2020 MetaMetrics Inc.

Implementation Phase

This lesson follows prior work with proportional and non-proportional linear equations. Before this problem is presented, students should be proficient in constructing a function to model a linear relationship between two quantities. Students should have worked with multiple representations of linear relationships, that is, with contexts, tables, graphs, and equations. They should be able to move easily from each of the representations to another. It is the first time that students have analyzed two linear relationships together in a system.

Day 1

- Give students a copy of the graphs and give them private think time to make observations.
- Discuss what students notice, reviewing key concepts about graphs and equations of lines.
- Present the problem. “When will the tanks hold the same amount of water?”
- Allow time for students to work on the problem with a partner.
- As partners finish the problem, give them extension problems.
- Analyze the work by discussing various pathways and extensions of the problem.

Day 1 Teacher Notes:

1) After giving students a copy of the graphs, read together the brief introduction about the two tanks.

2) Give students private think time to make observations about the graphs. Direct them to think about what they notice and what they wonder.
3) Have students share their observations, cautioning them not to give specifics that might answer questions about the problem. (The teacher might make use of a Think-Pair-Share protocol, allowing students to discuss with a partner before sharing with the class.) Students might notice:
    - There are two "lines" on this grid.
    - Tank T is losing water faster than Tank W is gaining water.
    - The lines will eventually cross.
    - The x-axis is numbered by ones; the y-axis by hundreds.

   Students might wonder:
    - Where will the lines cross?
    - When will Tank T be empty?
    - When will the Tanks have the same amount of water?

   The teacher should anticipate other student responses. This is an excellent time to draw out MHM 6, "Attend to Precision", by stressing the use of clear and precise language in the discussion.
   A list can be made of student "wonderings", as these can provide extensions to the problem.
4) **Remind** students that they have learned multiple representations of linear functions and that they should represent this problem in each of those ways. They should be able to explain how the solution is shown in each representation. **Stress this.**
5) **A**llow students time to work on the problem with a partner.
   Be watchful!
6) Be ready with the questions provided for struggling students and be on the lookout for misconceptions. In both cases ask probing questions that do not give away an answer or that specifically suggest a solution method.
7) Be ready to give extensions to early finishers.
8) Make observations about the various ways students are approaching the problem so that these can be drawn out in processing the lesson. Make decisions about the order in which you will have students present their solutions and methods.
9) Discuss the solution and points mentioned in the **Analyze** section.

Day 2 – as needed to continue working on/analyzing the problem

Day 2 Teacher Notes:
Discuss the solution and points mentioned in the **Analyze** section. It is important that there is time to process the lesson, even if there is a need for the discussion to continue a second day.

Share, Discuss, and Analyze Phase
Essential Understanding #1: *A linear function is defined by a rate of change that is constant. The function can be represented by a table, an equation, a graph, or in words.*
Share – The graph is given to students and they observe, then discuss what they notice and what they wonder. The problem is presented to the students.
Discuss – Since students are proficient in constructing a function to model a linear relationship between two quantities, they should be noticing and discussing the rate of change, the y-intercepts, the direction of the lines, and the fact that each situation has a constant rate of change.

Analyze –

1) Assuming that students approached the problem in several different ways, it is up to the teacher to decide a logical order in which to discuss the solution pathways. One suggestion is to begin with a pair who used the graphing method, but got an incorrect solution. Then look at a paper that has the correct solution and discuss how to be certain that the lines are in the correct place. (It is important to have a classroom climate in which learning from mistakes is honored.)
2) A logical next step might be to look at solutions from a table. Ask, "How do you know what your independent variable and dependent variable are?" How can you identify the rate of change from the table?" Some students may find it easy to find answers to the extension problems by using a table.
3) Finally, check if equations were written. What form of the equation was used? What is the most logical form to use in this situation? Since the starting amount (y-intercept) and the rate of change (the slope) can be found on the graphs, the slope-intercept form may be used, not simply because it is a student's preferred form, but because it is the most logical for this particular problem.
4) Discuss extension problems. Discuss which representation was used in addressing them. Appropriate questions to ask are those that require students to find an amount of water in a tank at a specific time, and the time when a specific amount of water is a tank. Observe to see if students are using both the equation and the graph to answer the question. (Ask, "Is there another way you can find that?") If students are only extending and reading the graph, ask, "How much water will be in Tank W at 20 hours?" Also ask, "Is it true that there will be 1000 gallons of water in Tank W at 16 hours?"* and "Is it true that there will be 925 gallons of water in the tank at 25 hours?", addressing the concept that the values must satisfy the equation and would lie on the line. *If a student rapidly responds, "yes" to this question, it may expose a misconception. Since there is a value, (8, 500) the student may believe that since the time doubles, so will the amount of water. This would only be true in a proportional situation. PLEASE NOTE – If students have had multiple prior experiences with questions like these, they do not need to be discussed here, however they should be addressed in depth somewhere in the curriculum.

Essential Understanding #2:

Systems of linear equations can be used to model and solve real-world problems.

Share- The graph is given to students and they observe, then discuss what they notice and what they wonder. The problem is presented to the students.

Discuss- Partners work together and should discuss how to find when tanks hold the same amount of water and how and they found it. They should discuss how they would find the answer to the question in the model they choose. If they find the solution using one model, they should check their work by finding it using another model.

Analyze- This analysis should take place as a part of the previous discussion. It should be mentioned here that this is a "system of equations", that we are looking representations for both tanks together.

1) When the class is looking at the graphs, ask the question, "How does the solution appear in the graph?" "What does this point of intersection mean?"
2) When looking at the tables, ask the question, "How can you find the solution in the table?"
3) When looking at the equations, ask, "How can you tell by looking at the equations that the lines are going to intersect? At the beginning of the lesson, one observation that students probably made was that the lines would intersect at some point. Now is the time to discuss why they intersect. (They have different slopes. Students might observe that one has a positive slope and the other a negative slope. Later they will discover that as long as the slopes are different, the

lines will intersect.) Ask the question and discuss how they might know this by only looking at the equations.

4) This task provides an opportunity to introduce students to the idea of solving the system symbolically. "Is there another way to find the solution? What is the purpose of writing the equations?" Since we are trying to find when x and y are equal for both tanks, and since $y = -50x + 900$ for Tank T and $y = 25x + 300$ for Tank W, we can lead students through the process of setting the expressions, $-50x + 900$ and $25x + 300$, equal to each other equal to each other and solving for x. This gives the value of x, when the y values are equal to each other. ($x = 8$) Then by substituting x into either or both of the equations, students will find that $y = 500$. Thus, the solution is (8, 500).
5) Finally ask, "Which model do you prefer and why?"
   Thinking Ahead: Eventually, students will learn why it is important to solve systems symbolically rather than graphically, and other methods for doing so. For instance, one method is to use the equations in standard form and combine the equations. It is an excellent idea to keep the work from this lesson and to revisit it when tackling these other methods.

Task In Action

The video clips below provide a demonstration of the task being implemented in a classroom as it aligns to the Effective Mathematics Teaching Practice indicated. These clips should be used by the teacher to model the implementation of the task in his or her own classroom.

- Use and Connect Mathematical Representations
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
  - Video Clip 4
  - Video Clip 5
- Facilitate Meaningful Mathematical Discourse
  - Video Clip