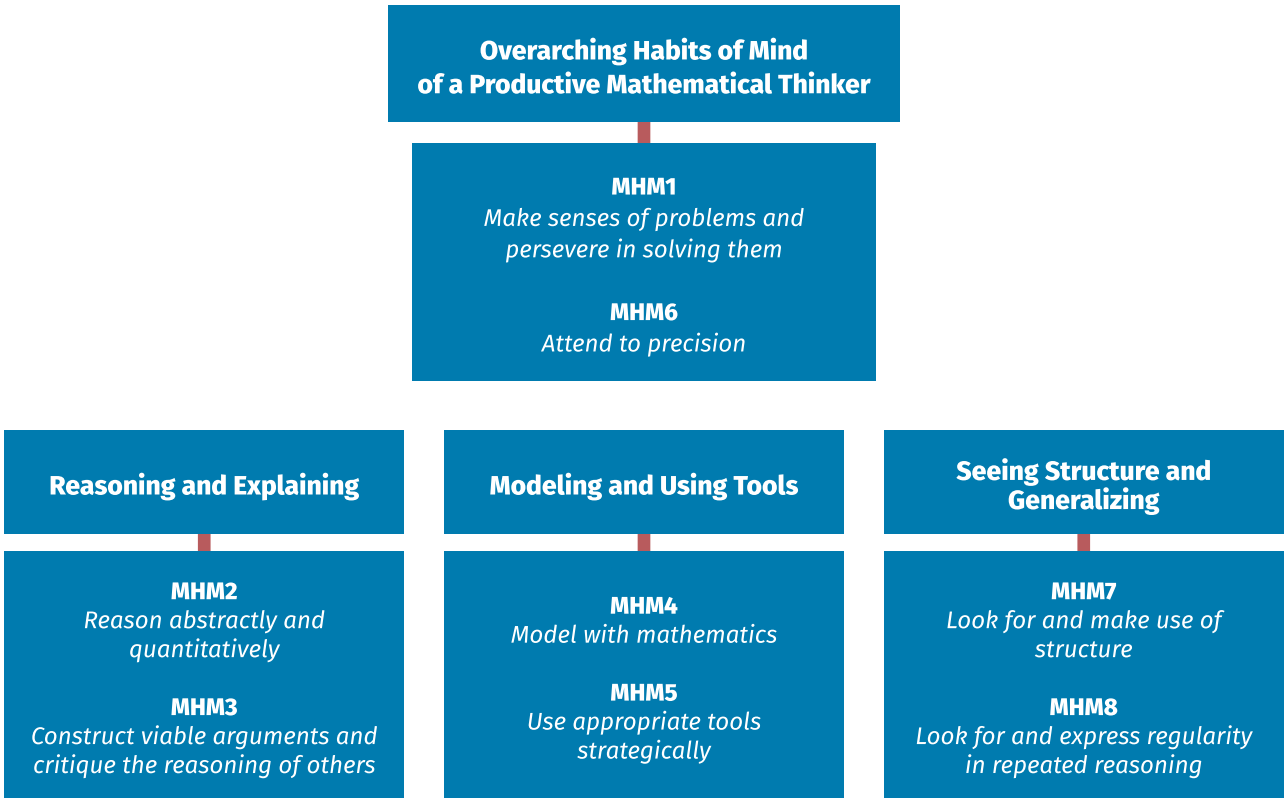
Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Financial Algebra/Mathematics

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in this course will focus on financial applications designed to deepen and extend understanding of mathematics. Students in Financial Algebra/Mathematics will communicate effectively, using accurate mathematical language in a financial context. Students will interpret and analyze various functions, graphs and data in order to make responsible and wise financial decisions in the context of their personal lives regarding banking services, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Mathematical Language in a Financial Context	The Algebra of Finance
• Demonstrate reasoning skills in developing, explaining, and justifying sound financial decisions. • Communicate effectively, using accurate mathematical language in a financial context.	• Apply algebraic skills and concepts to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.
Financial Modeling with Functions	**Modeling with Data**
• Interpret and analyze various functions, graphs, and data to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.	• Create, interpret, and evaluate financial models to make responsible and wise financial decisions in the context of their personal lives regarding banking services, consumer credit, automobile purchases and maintenance decisions, income tax and employee benefits, and business decisions.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Financial Algebra/Mathematics:

Mathematical Language in a Financial Context	
Communicate reasoning and decisions.	Standards 1-3
Algebra/Mathematics of Finance	
Use algebraic reasoning and techniques.	Standards 4-11
Financial Modeling with Functions	
Construct, graph, use, and interpret functions.	Standards 12-27
Financial Modeling with Data	
Represent, summarize, and evaluate data.	Standards 28-37

Mathematical Language in a Financial Context

Cluster	**Communicate reasoning and decisions.**
M.FAM.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical decision making (e.g., demonstrate reasoning skills in creating and presenting a budget of monthly expenses based on a career pathway income, and analyze the soundness of the mathematical reasoning of others; determine outlook for a chosen career pathway and use the average salary to determine if the desired cost of living can be met).
M.FAM.2	Communicate with and about mathematics in a financial context.
M.FAM.3	Communicate with and about mathematics in writing and orally, both independently and collaboratively, by preparing financial plans (e.g., plan for an emergency savings fund that will last three to six months in the case of loss of income; determine the total percentage of income paid to taxes or the percentage of total salary that a benefits package represents).

Algebra/Mathematics of Finance

Cluster	**Use algebraic reasoning and techniques.**
M.FAM.4	Interpret parts of an expression or equation, such as terms, factors, and coefficients, in a variety of financial models including those found in stock markets, automobile financing and in banking contexts.
M.FAM.5	Create and solve linear equations and inequalities in one variable and use them to solve problems in financial applications that may include, but are not limited to, stock markets, automobile ownership, business modeling, or employment (e.g., calculate wages by hourly rates or pay periods to make decisions about pay in a real world context).
M.FAM.6	Create equations in two or more variables to represent relationships between quantities in a financial context; graph equations on coordinate axes with labels and scales. Financial contexts may include, but are not limited to, stock markets, automobile ownership, business modeling employment, banking, consumer debt, and independent living decisions regarding taxes or planning for retirement (e.g., create a linear expense equation based on fixed and variable expenses and graph choosing an appropriate scale and origin for the graph).

Cluster	**Use algebraic reasoning and techniques.**
M.FAM.7	Represent constraints in financial applications by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context (e.g., create a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental; find the percentage of total salary that a benefits package represents; calculate taxes owed based on a given income and tax table and determine total percentage of income paid to taxes; calculate the gross pay and net pay using the FICA percentage (7.65%), retirement contribution, and worker's compensation insurance (employer match)).
M.FAM.8	Rearrange formulas for financial applications to highlight a quantity of interest, using the same reasoning as in solving equations. Know difference between growth and decay functions (e.g., solve the literal equation for exponential depreciation to find a depreciation rate and the literal equation for continuous interest to find the interest rate; apply the formula for average daily balance, (average daily balance*APR*days in billing cycle)/365, using literal equations with varying APRs and billing cycles).
M.FAM.9	Solve systems of linear equations exactly and approximately (e.g., with graphs) in making financial decisions, focusing on pairs of linear equations in two variables (e.g., create and solve a system of equations based on the expenses incurred and monthly payment when choosing home ownership versus rental).
M.FAM.10	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials (e.g., combine the polynomials that model income and expense to create a profit model).
M.FAM.11	Solve quadratic equations in one variable in a financial context that may include, but are not limited to, business modeling or employment decisions (e.g., given a quadratic equation that models a profit function, determine the break-even points; apply braking distance/stopping distance formulas to solve problems related to driving and safety data).

Cluster	**Construct, graph, use, and interpret functions.**
M.FAM.12	Use functions to model financial situations. Use multiple representations of functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically (e.g., develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; use linear and polynomial functions to evaluate and communicate quantities as required by Internal Revenue Service and Social Security Administration regulations and to determine when and why the models may be discontinuous).
M.FAM.13	Use function notation in financial applications, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a financial context (e.g., in making decisions regarding retirement income, apply the formula $A(t) = Pe^{rt}$ to determine future value).
M.FAM.14	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Use this relationship in analyzing financial situations (e.g., compare the linear function modeling simple interest with the exponential function modeling compound interest).
M.FAM.15	Select a function that models a relationship between two quantities in financial contexts, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship (e.g., write, graph, and interpret the revenue (quadratic) function in comparison to the expense (linear) function using key features of the functions; reason quantitatively to compare subsidized and unsubsidized loans, as well as other forms of financial aid available to college students; calculate mortgage payments, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage).
M.FAM.16	Interpret the parameters in a linear or exponential function in terms of a context (e.g., investigate and compare, using technology and regression, historical data to determine if automobile depreciation follows a linear or exponential model).
M.FAM.17	Construct linear and exponential functions modeling financial contexts, including arithmetic and geometric sequences, given a graph, a description of a relationship, or given input-output pairs including reading these from a table (e.g., utilize linear and exponential functions to compare simple with compound interest).
M.FAM.18	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.19	Calculate and interpret the average rate of change of a function modeling a financial context (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph (e.g., examine depreciation trends).

Cluster	**Construct, graph, use, and interpret functions.**
M.FAM.20	Graph functions expressed symbolically and show key features of the graph (e.g., graph the linear, quadratic, or exponential curve that models the demand versus supply functions and find the equilibrium point with and without technology). Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.
M.FAM.21	Compare properties of two functions each represented in a different way, such as algebraically, graphically, numerically in tables, or by verbal descriptions (e.g., utilize linear and exponential functions to compare simple with compound interest; calculate and compare using both the loan payment formula and payment schedules in table format, the monthly cost of purchasing an automobile, and discuss the feasibility of that payment in relation to monthly budget; compare two functions showing interest accrued when paying the minimum monthly payment over time compared to paying a larger monthly payment; identify and compare the average rate of change between given time periods).
M.FAM.22	Graph linear and quadratic functions and show intercepts, maxima, and minima (e.g., in the model of a profit function, determine the break-even points, the maximum possible loss, and the maximum profit).
M.FAM.23	Write a function that describes a relationship between two quantities in a financial context (e.g., calculate the costs associated with purchasing a vehicle, including leasing, purchasing with cash, or with a loan).
M.FAM.24	Identify the effect on functions that model financial situations of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology (e.g., identify the impact of a change in a constraint in a function that models retirement planning, business income and expenses, or employment benefits).
M.FAM.25	Graph square root, cube root, and piecewise-defined functions that model financial situations, including step functions and absolute value functions (e.g., develop and communicate the appropriateness of representing a commission salary using a linear versus a piecewise function; analyze graphs of functions that model profit).
M.FAM.26	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model financial situations, and translate between the two forms (e.g., create recursive and explicit models of sequences related to retirement planning, amortization schedules for a loan, comparing subsidized and unsubsidized loans, reasoning and making decisions about the length of the loan and a fixed versus adjustable rate mortgage).
M.FAM.27	Apply exponential formulas to solve for future and present value of investments by hand or with graphing technology (e.g., $PV=FV*(1/(1+r)^n)$ and $A(t) = Pe^{rt}$).

Cluster	**Represent, summarize, and evaluate data.**
M.FAM.28	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots). Instructional Note: Data may address, but is not limited to automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.29	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit regression lines to scatterplots and make predictions based on lines of best fit. Find and interpret correlation coefficients of regression equations in financial situations (e.g., use scatter plots to show correlation between two funds, two stocks or even a stock and the general market, or in business situations to forecast sales or to compare revenue to the number of units sold).
M.FAM.30	Create a data display modeling financial situations. Instructional Note: This may include, but is not limited to, modeling the different savings options for a given investment at local banking establishments; calculating and comparing the monthly cost of purchasing an automobile using both the loan payment formula and payment schedules in table format; creating an amortization schedule through the use of spreadsheet technology and the formula tool for a loan given principle, term, monthly payment, and interest rate; creating representations of pay schedules using a variety of modeling technologies; and making decisions in a financial context based on those representations.
M.FAM.31	Summarize categorical data in various forms (e.g., two-way frequency tables, circle graphs, segmented bar charts). Interpret relative frequencies in the context of the data in making financial decisions.
M.FAM.32	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: Data sets may address, but are not limited to, automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning.
M.FAM.33	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays (e.g., use units appropriately as a way to understand multi-step problems in relationship to understanding credit card fees and finance charges; compute distance, rate and time to solve problems to analyze driving and safety data, using single and multiple unit conversion; use and compare researched reaction times and vehicle velocity, as well as accepted equations to solve problems with braking distances).
M.FAM.34	Use financial models from automobile financing, investing in the stock market, business, employment, banking, consumer credit, taxes, and retirement planning to solve problems.
M.FAM.35	Evaluate reports based on data. Data may address, but is not limited to, planning for retirement or stock markets.

Cluster	**Represent, summarize, and evaluate data.**
M.FAM.36	Use probability and expected value to analyze financial situations (e.g., model and compare automobile insurance policies).
M.FAM.37	Evaluate the impact of taxes on business ownership including property tax, sales tax, social security, retirement, and disability benefits. Evaluate the impact of taxes on personal finance decisions.